LECTURE 2.1

The Real Business Cycle Model

ECON42240 Advanced Macroeconomics

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Spring 2027

Where We Are

Last Week's Lecture

Before the class test, Lecture 1.5 solved linear models under rational expectations:

1. **Rational Expectations.**
   The assumption, and the forward solution it delivers.
2. **Reduced Forms and Persistence.**
   A solved model as a VAR, and lagged dynamics.
3. **Solving the System.**
   Eigenvalues, the Blanchard–Kahn count, and generalised Schur.
4. **The Lucas Critique.**
   Why reduced-form coefficients shift when policy changes.

Lecture Plan

1. **The Planner’s Problem.**
   One planner, one technology shock, no frictions.

2. **The First-Order Conditions.**
   From a Lagrangian to six equations.

3. **Log-Linearising.**
   Around a non-stochastic steady state.

4. **What the RBC Model Gets Wrong.**
   Money, propagation, and hours worked.

SECTION 1

The Planner's Problem

The Model the Solver Cannot Yet Take

Today's model must reach the form the solver takes (Lecture 1.5):

$$KZ_t = AZ_{t-1} + B\,E_t Z_{t+1} + HX_t \qquad (1)$$

An Optimising Model Is Not Linear

The solver takes only equations linear in Z_t. First-order conditions carry products, powers and ratios.

Kydland and Prescott Built the First DSGE Model

Kydland and Prescott built a cycle from optimising households and clearing markets (Kydland & Prescott, 1982):

- **What Is Inside:** Preferences, a technology, a resource constraint, and nothing else.
- **What Is Outside:** Nominal prices and wages, money, government, market imperfections.
- **Why It Endures:** Later models keep the method and change the assumptions.

What the Social Planner Maximises

A planner picks everyone's consumption and hours, with a discount factor (β):

$$\max \; E_t \left[\sum_{i=0}^{\infty} \beta^{\,i} \big(U(C_{t+i}) - V(N_{t+i}) \big) \right] \tag{2}$$

where:

- C_t: consumption
- N_t: hours worked
- U: utility of consumption
- V: disutility of hours

> **Capital Letters Mark Levels**
> A capital letter is a level here. Lower case takes a new meaning in the third section.

The Resource and Capital Constraints

Capital and labour produce output, with the capital share (α) as capital's exponent:

$$Y_t = C_t + I_t = A_t K_{t-1}^{\alpha} N_t^{1-\alpha}, \qquad K_t = I_t + (1-\delta)K_{t-1} \tag{3}$$

where:

- Y_t: output
- I_t: investment
- K_t: capital stock
- A_t: total factor productivity
- δ: depreciation rate

Capital Is Ready a Period Early
Output at t uses K_{t-1}. Capital installed today produces from tomorrow, which makes the model dynamic.

Technology Is the Only Source of Shocks

Only technology is random, an AR(1) in logs around a long-run level (A^*):

$$\log A_t = (1 - \rho) \log A^* + \rho \log A_{t-1} + \epsilon_t \tag{4}$$

> **One Shock Has to Explain Every Cycle**
> Every fluctuation traces back to ϵ_t, a surprise in what inputs produce. Demand, policy and finance have no term.

A Planner's Solution Is the Market Outcome

With perfect competition and instant market clearing, one planner reproduces the market:

- **The Shortcut:** Solve one optimisation instead of a household's and a firm's.
- **What Justifies It:** Perfect competition, no externalities, and complete markets.

> **Imperfect Competition Breaks the Shortcut**
> Firms and households then need separate conditions. Lecture 2.3 starts there.

The RBC Model as a Benchmark for Others

Critics reject optimising agents and clearing markets. Whelan answers that the assumption is the test:

> **A Baseline Is Judged by What It Leaves Unexplained**
> If optimisation and clearing markets produce a cycle, sticky prices are not needed. If not, the gap names what a richer model needs.

The last section judges the model against evidence it was never fitted to.

Technology Shocks Face a Credibility Objection

Recessions here are the optimal response to technology shocks. Whelan puts the objection and the defence:

- **The Objection:** Nobody reads about the technological regress behind a recession.
- **The Defence:** TFP drives long-run growth per hour, and need not grow steadily.

> **An Elasticity Above One Avoids Outright Regress**
> At least in theory, then, recessions need no fall in technology. Today's model gives output an impact elasticity of 2.1.

How Many First-Order Conditions Does the Planner Need?

The planner maximises an expected, discounted sum over an infinite future.

> **Question**
> Every future date carries its own choices. Why is one set of first-order conditions enough?

Take answers from the room.

SECTION 2

The First-Order Conditions

Taking the Expectation Through a Derivative

A sum over states shows why a derivative passes inside an expectation:

$$G(x) = \sum_{k=1}^{N} p_k F(a_k, x) \qquad \Longrightarrow \qquad G'(x) = \sum_{k=1}^{N} p_k F'(a_k, x) = E_t F'(x) \tag{5}$$

where:

- G: expected payoff
- F: payoff in one state
- a_k: state k
- p_k: probability of state k

Differentiate Inside the Expectation

The conditions for maximising $E_t F(x)$ are $E_t F'(x) = 0$. Every first-order condition today uses that step.

Combining the Two Constraints into One

The capital equation gives investment as capital chosen less capital surviving:

$$I_t = K_t - (1-\delta)K_{t-1} \tag{6}$$

Investment sits in both constraints and not the objective, so substitute it out:

$$A_t K_{t-1}^{\alpha} N_t^{1-\alpha} = C_t + K_t - (1-\delta)K_{t-1} \tag{7}$$

The Lagrangian for the Planner's Problem

Each period's constraint gets a Lagrange multiplier (λ_t), added to the objective:

$$\mathcal{L} = E_t \sum_{i=0}^{\infty} \beta^i \Big[U(C_{t+i}) - V(N_{t+i}) + \lambda_{t+i} \big(A_{t+i} K_{t+i-1}^{\alpha} N_{t+i}^{1-\alpha} + (1-\delta) K_{t+i-1} - K_{t+i} - C_{t+i} \big) \Big] \tag{8}$$

where:

- $\mathcal{L}$: Lagrangian

The Multiplier Prices One Unit of Output
λ_t is the utility one more unit of output at t buys. The same letter was 1.1's smoothing weight and 1.5's root.

Only Two Blocks Contain the Variables Dated t

Only two blocks of the infinite sum contain a variable dated t:

$$U(C_t) - V(N_t) + \lambda_t \left(A_t K_{t-1}^{\alpha} N_t^{1-\alpha} - C_t - K_t + (1-\delta) K_{t-1}\right) + \beta E_t \left[\lambda_{t+1}\left(A_{t+1} K_t^{\alpha} N_{t+1}^{1-\alpha} + (1-\delta) K_t\right)\right] \quad (9)$$

One Set of Conditions Covers Every Date

Variables dated $t + n$ enter as those dated t do, discounted by β^n. Differentiating once is enough.

The Conditions for Consumption and for Capital

Output consumed today is worth its marginal utility, which the multiplier must equal:

$$\frac{\partial \mathcal{L}}{\partial C_t} : \; U'(C_t) - \lambda_t = 0 \tag{10}$$

Saving a unit as capital costs λ_t now and pays back tomorrow:

$$\frac{\partial \mathcal{L}}{\partial K_t} : \; -\lambda_t + \beta E_t \left[\lambda_{t+1} \left(\alpha \frac{Y_{t+1}}{K_t} + 1 - \delta \right) \right] = 0 \tag{11}$$

The Conditions for Hours and the Constraint

An hour's disutility equals its marginal product, the competitive real wage, valued at λ_t:

$$\frac{\partial \mathcal{L}}{\partial N_t} : \ -V'(N_t) + (1-\alpha)\lambda_t \frac{Y_t}{N_t} = 0 \tag{12}$$

Differentiating by the multiplier returns the combined constraint:

$$\frac{\partial \mathcal{L}}{\partial \lambda_t} : \ A_t K_{t-1}^{\alpha} N_t^{1-\alpha} - C_t - K_t + (1-\delta)K_{t-1} = 0 \tag{13}$$

The Marginal Value of a Unit of Capital

Capital earns a gross return, its marginal product plus the surviving $1 - \delta$:

$$R_{t+1} \equiv \alpha \frac{Y_{t+1}}{K_t} + 1 - \delta \tag{14}$$

With the bracket named R_{t+1}, adding λ_t to both sides gives:

$$\lambda_t = \beta E_t \left(\lambda_{t+1} R_{t+1} \right) \tag{15}$$

where:

- R_{t+1}: gross return on capital

The Keynes–Ramsey Condition for Consumption

The consumption condition holds at every date, so U' replaces λ twice:

$$U'(C_t) = \beta E_t \left[U'(C_{t+1}) R_{t+1} \right] \tag{16}$$

Consumption Today Trades Against Consumption Tomorrow
The planner gives up consumption now and gets it back at capital's return.

Reading the Keynes–Ramsey Condition

The condition swaps a small amount of consumption (Δ) between two dates:

- **Give Up Δ Today:** Consumption falls, at a utility cost of $U'(C_t)\Delta$.
- **Invest It:** Δ returns $R_{t+1}\Delta$, worth $\beta E_t[U'(C_{t+1})R_{t+1}\Delta]$ today.
- **Along the Best Path:** The two sides are equal, so no swap helps.

No Small Swap Improves the Optimal Path

If $U'(C_t)$ exceeded the right side, consuming more today would raise utility.

CRRA Utility with Separable Labour

Only some utility functions allow steady growth; here each hour costs a disutility (a):

$$U(C_t) - V(N_t) = \frac{C_t^{1-\eta}}{1-\eta} - aN_t \tag{17}$$

where:

- η: CRRA curvature

> **The Same Curvature at Every Level**
> η is the elasticity of marginal utility, whatever consumption is. The 2015 paper writes it θ.

The Two Optimality Conditions Under CRRA

Marginal utility is now $C_t^{-\eta}$, so the Keynes–Ramsey condition reads:

$$C_t^{-\eta} = \beta E_t \left[C_{t+1}^{-\eta} R_{t+1} \right] \tag{18}$$

An hour's valued product equals its cost a, which fixes output per hour:

$$\frac{Y_t}{N_t} = \frac{a}{1-\alpha} C_t^{\eta} \tag{19}$$

Consumption Alone Pins Output per Hour

Given consumption, the real wage is fixed. Higher consumption raises that wage and cuts hours.

Three Constraints Hold in Every Period

Three constraints tie output to its production and its split:

$$\begin{aligned} Y_t &= C_t + I_t && \text{resource constraint} \\ Y_t &= A_t K_{t-1}^{\alpha} N_t^{1-\alpha} && \text{production function} \\ K_t &= I_t + (1-\delta) K_{t-1} && \text{capital accumulation} \end{aligned} \tag{20}$$

A Definition and Two Choices Complete the Model

A return definition and two optimality conditions complete the six equations:

$$
\begin{aligned}
R_t &= \alpha \frac{Y_t}{K_{t-1}} + 1 - \delta && \text{return on capital} \\
C_t^{-\eta} &= \beta E_t \left[C_{t+1}^{-\eta} R_{t+1} \right] && \text{Euler equation} \\
\frac{Y_t}{N_t} &= \frac{a}{1-\alpha} C_t^{\eta} && \text{labour condition}
\end{aligned}
\tag{21}
$$

Which of These Equations Is Not Linear?

Six equations now determine six variables, given technology. The solver takes only linear ones.

> **Question**
> Which of the six equations are already linear? What makes each of the rest fail?

Take answers from the room.

SECTION 3

Log-Linearising

Approximating a Non-Linear Function by a Line

A first-order Taylor expansion around a chosen point (x^*, y^*) is linear:

$$F(x_t, y_t) \approx F(x^*, y^*) + F_x(x^*, y^*)(x_t - x^*) + F_y(x^*, y^*)(y_t - y^*) \tag{22}$$

A one per cent deviation, squared, is one part in ten thousand.

> **Accuracy Fails Far from the Point**
> Second and higher powers are dropped because they are small near (x^*, y^*). Far from it they are not small.

Taking Logs Before Linearising

Every non-linear equation failed through a product, a power or a ratio. Logs turn all three into sums:

- **The Point Chosen:** A steady-state path where real variables share one growth rate.
- **Why That Point:** The economy fluctuates around the path, so deviations stay small.
- **What Comes Out:** Linear equations in log deviations from steady-state values.

> **A Steady State Is a Path or a Point**
> The economy settles at its steady state when no shocks arrive. Today's version has no growth, so it is constant.

Log Deviations Read as Percentages

A difference in logs is approximately a percentage difference:

$$\log X - \log Y \approx \frac{X - Y}{Y} \tag{23}$$

Output 2 per cent above its steady state gives $\log 1.02 = 0.0198$.

Every Response Comes Out in per Cent

Each variable is a percentage deviation; each coefficient is an elasticity. The system is the model's business-cycle component.

Writing a Variable as a Deviation from Steady State

The log deviation (x_t) is the log less the log of the steady-state value (X^*):

$$x_t \equiv \log X_t - \log X^* \tag{24}$$

Adding $\log X^*$, exponentiating, then using $e^{x_t} \approx 1 + x_t$:

$$\log X_t = \log X^* + x_t \implies X_t = X^* e^{x_t} \approx X^*(1 + x_t) \tag{25}$$

Lower Case Now Means a Log Deviation

In Lecture 1.1 lower case marked a log. From here it marks a log deviation from the steady state.

Products of Two Small Deviations Are Dropped

Writing each level of a product around its steady state:

$$X_t Y_t \approx X^* Y^* (1 + x_t)(1 + y_t) \quad \textbf{(26)}$$

Multiplying out leaves a term in $x_t y_t$, far smaller than either deviation, so it is dropped:

$$X_t Y_t \approx X^* Y^* \left(1 + x_t + y_t + \cancel{x_t y_t}\right) = X^* Y^* (1 + x_t + y_t) \quad \textbf{(27)}$$

A Power of a Level Multiplies Its Deviation

Raising a level to a power (ω) raises both factors of $X^* e^{x_t}$ to it:

$$X_t^{\omega} = \left(X^* e^{x_t}\right)^{\omega} = \left(X^*\right)^{\omega} \left(e^{x_t}\right)^{\omega} \tag{28}$$

A power of an exponential multiplies the exponent; then $e^{\omega x_t} \approx 1 + \omega x_t$:

$$X_t^{\omega} = \left(X^*\right)^{\omega} e^{\omega x_t} \approx \left(X^*\right)^{\omega} \left(1 + \omega\, x_t\right) \tag{29}$$

where:

- ω: any exponent

No Trend Growth Makes Every Steady State Constant

Technology has no drift, so nothing grows in the long run:

$$a_t = \rho a_{t-1} + \epsilon_t \tag{30}$$

> **Zero Growth Is Chosen for Shorter Algebra**
> The model describes fluctuations around a flat path. A growing steady state works too, with extra bookkeeping.

Example One: Three Levels in the General Form

Each level in the resource constraint has the general deviation form:

$$X_t = X^* e^{x_t} \tag{31}$$

The resource constraint is three such levels, one sum:

$$Y_t = C_t + I_t \tag{32}$$

Each Level Brings Its Own Steady State

X_t is Y_t, then C_t, then I_t. Each takes its own X^* and its own x_t.

Example One: Substituting, Then Approximating

Dropping each match into the constraint is exact:

$$Y^* e^{y_t} = C^* e^{c_t} + I^* e^{i_t} \tag{33}$$

Replacing each exponential by one plus its exponent is the only approximation:

$$Y^*(1 + y_t) \approx C^*(1 + c_t) + I^*(1 + i_t) \tag{34}$$

Example One: Multiplying Out Shows the Identity

Multiplying out each bracket separates the constants from the deviations:

$$\boxed{Y^*} + Y^* y_t \approx \boxed{C^* + I^*} + C^* c_t + I^* i_t \tag{35}$$

Steady-State Values Satisfy the Constraint Too

With no shock, $Y^* = C^* + I^*$. The two marked terms are equal.

Example One: Steady-State Shares Weight the Deviations

Subtracting the identity from both sides leaves only deviations:

$$Y^* y_t \approx C^* c_t + I^* i_t \tag{36}$$

Dividing both sides by Y^* turns each weight into a share of output:

$$y_t \approx \frac{C^*}{Y^*} c_t + \frac{I^*}{Y^*} i_t \tag{37}$$

Example Two: Each Factor Is a Level Raised to a Power

The power rule covers any level raised to any exponent:

$$X_t^{\omega} = \left(X^*\right)^{\omega} e^{\omega x_t} \tag{38}$$

The production function multiplies three such powers together:

$$Y_t = A_t \, K_{t-1}^{\alpha} \, N_t^{1-\alpha} \tag{39}$$

Each Factor Brings Its Own Exponent
$\omega = 1$ for technology, $\omega = \alpha$ for capital and $\omega = 1 - \alpha$ for hours.

Example Two: Each Factor Written Around Its Steady State

Each factor takes the rule's exact form, with its own exponent dropped in:

$$Y^* e^{y_t} = A^* e^{a_t} \left(K^*\right)^{\alpha} e^{\alpha k_{t-1}} \left(N^*\right)^{1-\alpha} e^{(1-\alpha) n_t} \tag{40}$$

Nothing Is Approximated Yet

Only exact rules for powers were used. Output on the left is the rule with $\omega = 1$.

Example Two: Cancelling the Steady-State Identity

Gathering the steady-state factors at the front puts output's identity in view:

$$Y^* e^{y_t} = \boxed{A^* \left(K^*\right)^{\alpha} \left(N^*\right)^{1-\alpha}} e^{a_t} e^{\alpha k_{t-1}} e^{(1-\alpha) n_t} \tag{41}$$

The marked product is Y^*, so dividing both sides by Y^* leaves:

$$e^{y_t} = e^{a_t} e^{\alpha k_{t-1}} e^{(1-\alpha) n_t} \tag{42}$$

Example Two: Multiplying Out the Three Brackets

Replacing each exponential by one plus its exponent is the one approximation:

$$1 + y_t \approx (1 + a_t)\big(1 + \alpha k_{t-1}\big)\big(1 + (1 - \alpha)n_t\big) \tag{43}$$

Multiplying out gives the three deviations and four products, each dropped:

$$\begin{aligned} 1 + y_t \approx\ & 1 + a_t + \alpha k_{t-1} + (1 - \alpha)n_t \\ & + \cancel{\alpha a_t k_{t-1} + (1 - \alpha)a_t n_t + \alpha(1 - \alpha)k_{t-1}n_t + \alpha(1 - \alpha)a_t k_{t-1}n_t} \end{aligned} \tag{44}$$

Example Two: Each Exponent Is an Elasticity

Subtracting one from both sides leaves the log-linear production function:

$$y_t \approx a_t + \alpha k_{t-1} + (1 - \alpha) n_t \tag{45}$$

The Capital Share Is an Elasticity

One per cent more capital raises output by α per cent, about a third.

The Three Constraints in Log Deviations

The same three constraints, now in log deviations:

$$
\begin{aligned}
y_t &= \frac{C^*}{Y^*} c_t + \frac{I^*}{Y^*} i_t && \text{resource constraint} \\
y_t &= a_t + \alpha k_{t-1} + (1 - \alpha) n_t && \text{production function} \\
k_t &= \frac{I^*}{K^*} i_t + (1 - \delta) k_{t-1} && \text{capital accumulation}
\end{aligned}
\tag{46}
$$

Examples one and two gave the first two.

The Log-Linear Behavioural Conditions

The three behavioural conditions, in log deviations:

$$
\begin{aligned}
n_t &= y_t - \eta c_t && \text{labour condition} \\
c_t &= E_t c_{t+1} - \frac{1}{\eta} E_t r_{t+1} && \text{Euler equation} \\
r_t &= \frac{\alpha}{R^*} \frac{Y^*}{K^*} (y_t - k_{t-1}) && \text{return on capital}
\end{aligned}
\tag{47}
$$

A Higher Expected Return Postpones Consumption

A higher $E_t r_{t+1}$ lowers c_t against $E_t c_{t+1}$. The factor $1/\eta$ sets by how much.

Three Coefficients Still Need the Steady State

The seventh equation is technology, $a_t = \rho a_{t-1} + \epsilon_t$, and three equations carry unknowns:

> **The Steady-State Ratios Are Still Unknown**
> The resource, capital and return equations carry C^*/Y^*, I^*/K^* and $\alpha Y^*/(R^*K^*)$.

The steady state supplies all three.

The Steady-State Euler Equation

Dividing the Keynes–Ramsey condition by $C_t^{-\eta}$, known at t, puts consumption in a ratio:

$$1 = \beta E_t \left[\frac{C_{t+1}^{-\eta}}{C_t^{-\eta}} R_{t+1} \right] \tag{48}$$

A negative power of a ratio is the flipped ratio's positive power:

$$1 = \beta E_t \left[\left(\frac{C_t}{C_{t+1}} \right)^{\eta} R_{t+1} \right] \tag{49}$$

The Return on Capital in Steady State

In steady state $C_t = C_{t+1} = C^*$ and $R_{t+1} = R^*$, with nothing uncertain:

$$1 = \beta \left(\frac{C^*}{C^*} \right)^{\eta} R^* = \beta R^* \tag{50}$$

Dividing both sides by β:

$$R^* = \beta^{-1} \tag{51}$$

Patience Alone Fixes the Steady-State Return

At $\beta = 0.99$, $R^* = 1.0101$: a net return of one per cent a quarter. No other parameter appears.

The Steady-State Output–Capital Ratio

The return's definition holds in steady state, with $R^* = \beta^{-1}$ on the left:

$$\beta^{-1} = \alpha \frac{Y^*}{K^*} + 1 - \delta \tag{52}$$

Subtracting $1 - \delta$ from both sides, then dividing by α:

$$\beta^{-1} - 1 + \delta = \alpha \frac{Y^*}{K^*} \quad \Longrightarrow \quad \frac{Y^*}{K^*} = \frac{\beta^{-1} + \delta - 1}{\alpha} \tag{53}$$

The Marginal Product Covers Waiting and Depreciation

It pays the reward for waiting, $\beta^{-1} - 1$, plus depreciation, δ. More of either needs a higher Y^*/K^*.

The Coefficient in the Return Equation

Both steady-state results drop in, and dividing by R^* multiplies by β:

$$\frac{\alpha}{R^*}\frac{Y^*}{K^*} = \alpha\,\beta\left(\frac{\beta^{-1}+\delta-1}{\alpha}\right) \tag{54}$$

The αs cancel, β multiplies in, and β is factored back out:

$$\frac{\alpha}{R^*}\frac{Y^*}{K^*} = \beta\left(\beta^{-1}+\delta-1\right) = 1+\beta\delta-\beta = 1-\beta(1-\delta) \tag{55}$$

The Capital Share Cancels Out

At $\beta = 0.99$ and $\delta = 0.015$ the coefficient is 0.025, so the return barely rises with output.

Investment Replaces Depreciated Capital

In steady state capital is constant, so the accumulation identity holds at K^*:

$$K^* = I^* + (1 - \delta)K^* \tag{56}$$

Subtracting $(1 - \delta)K^*$ from both sides, then dividing by K^*:

$$\delta K^* = I^* \qquad \Longrightarrow \qquad \frac{I^*}{K^*} = \delta \tag{57}$$

At $\delta = 0.015$, investment is 1.5 per cent of capital each quarter.

The Investment and Consumption Shares of Output

Dividing the investment–capital ratio by the output–capital ratio gives the last two coefficients:

$$\frac{I^*}{Y^*} = \frac{\alpha\delta}{\beta^{-1} + \delta - 1}, \qquad \frac{C^*}{Y^*} = 1 - \frac{\alpha\delta}{\beta^{-1} + \delta - 1} \tag{58}$$

0.20

of output is investment, at $\alpha = 1/3$, $\beta = 0.99$, $\delta = 0.015$

3.3 years

of output is the capital stock, at the same parameters

The Final System, Ready for the Solver

The steady-state coefficients (marked) drop in; seven variables fill Z_t, the shock X_t:

$$
\begin{aligned}
y_t &= \left(1 - \frac{\alpha\delta}{\beta^{-1}+\delta-1}\right) c_t + \frac{\alpha\delta}{\beta^{-1}+\delta-1}\, i_t && \text{resource constraint} \\
y_t &= a_t + \alpha k_{t-1} + (1-\alpha) n_t && \text{production function} \\
k_t &= \delta\, i_t + (1-\delta) k_{t-1} && \text{capital accumulation} \\
n_t &= y_t - \eta c_t && \text{labour condition} \\
c_t &= E_t c_{t+1} - \frac{1}{\eta} E_t r_{t+1} && \text{Euler equation} \\
r_t &= \left(1 - \beta(1-\delta)\right)(y_t - k_{t-1}) && \text{return on capital} \\
a_t &= \rho a_{t-1} + \epsilon_t && \text{technology}
\end{aligned}
\tag{59}
$$

THE SOLVER'S FORM

Where Would This Approximation Fail?

The system is a first-order expansion around a path the economy sits near.

> **Question**
> Name an episode where expanding around the steady state would mislead. Say why.

Take answers from the room.

SECTION 4

What the RBC Model Gets Wrong

Nothing in the Model Is Nominal

No price, nominal wage or quantity of money appears in the seven equations:

- **What Follows:** Money is neutral, so monetary policy changes no real quantity.
- **Fiscal Policy:** Added government spending would bring Ricardian equivalence.

> **Complete Monetary Neutrality Is a Prediction**
> Many people think monetary policy is crucial to the macroeconomy. The RBC model gives it no role.

Parameters for a Quarterly Economy

Running the model needs five numbers, and Whelan's are chosen for quarterly data:

$$\alpha = \tfrac{1}{3}, \qquad \beta = 0.99, \qquad \delta = 0.015, \qquad \rho = 0.95, \qquad \eta = 1 \tag{60}$$

Simulations here draw ϵ_t with standard deviation 0.31, as Whelan's program does.

Lecture 1.5 taught the Lucas critique.

Deep Parameters Are Meant to Survive Policy

Tastes and technology are assumed not to change when policy does. Lucas set that invariance as the condition for policy analysis (Lucas, 1976).

Simulated Cycles in Output, Consumption and Investment

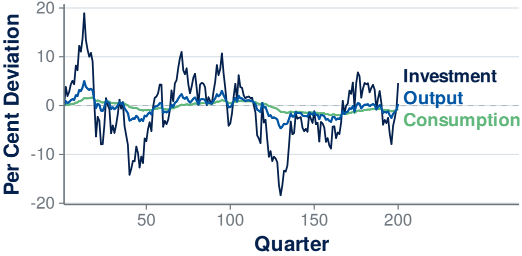

Figure 1: Simulated output, consumption and investment, 200 quarters, per cent deviations.

> **Investment Fluctuates Far More Than Consumption**
> Standard deviations: investment 5.8 per cent, consumption 0.9. Nothing was fitted to that ordering.

What the Simulation Got Right

Early RBC research counted results like these as a major strength:

- **Realistic Cycles:** Reasonable parameters roughly match observed fluctuations.
- **The Right Ordering:** Investment swings far further than consumption, as in the data.
- **The Claim Made:** Many economists said market imperfections were unnecessary.

> **A Frictionless Model Produced a Plausible Cycle**
> Nothing sticky, nominal or imperfect appears anywhere. The cycles came out anyway.

The Propagation Mechanisms Are Weak

Early advocates claimed the model's own dynamics amplified technology shocks into cycles:

- **The Claim:** Capital accumulation and extra hours would amplify and spread a shock.
- **The Finding:** Simulated output follows technology closely, quarter by quarter.

The Persistence Is Inherited, Not Produced

Output inherits its persistence from ρ in the technology process. The model supplies almost none of its own.

Simulated Output Follows Technology Closely

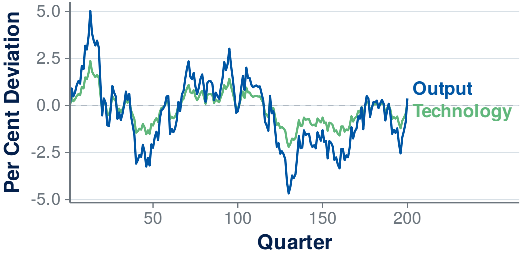

Figure 2: Simulated output and the technology that drove it, 200 quarters, per cent.

> **Output Tracks Technology Quarter by Quarter**
> Output and technology correlate at 0.9999; output swings twice as far. Both have an autocorrelation of 0.92.

Output Growth Is Positively Autocorrelated

Cogley and Nason found a fact the model does not match (Cogley & Nason, 1995):

- **In the Data:** Output growth in one quarter predicts growth in the next.
- **How Strong:** Small but significant: Whelan reports their coefficient as 0.34.
- **In the Model:** Simulated output growth does not show that pattern.

> **The Model Matches the Fact Only by Assumption**
> A technology process with autocorrelated growth reproduces the fact. The model does not generate it.

Simulated Growth Is Not Positively Autocorrelated

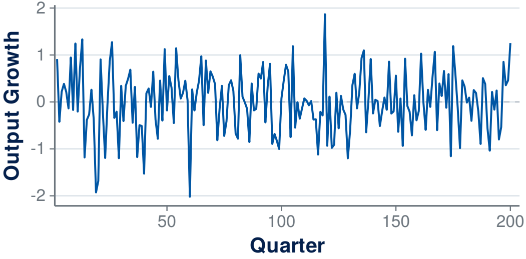

Figure 3: Simulated quarterly output growth, 200 quarters, per cent.

> **Simulated Growth Shows No Positive Autocorrelation**
> Its first-order autocorrelation is -0.05, against the 0.34 Whelan reports from Cogley and Nason.

Matching the Data Needs a Hump-Shaped Response

Cogley and Nason traced the failure to the shape of the responses:

> **A Hump Rises Before It Falls**
> A hump-shaped response rises for several quarters, then decays. Growth follows growth, which is positive autocorrelation.

The RBC model's output response is largest on impact, then decays.

Responses to a Technology Shock

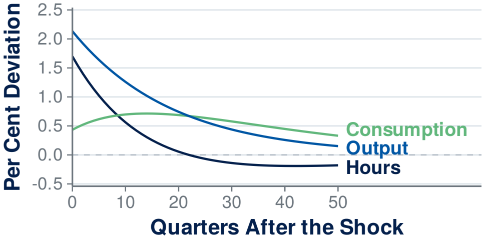

Figure 4: Output, consumption and hours after a one per cent technology shock.

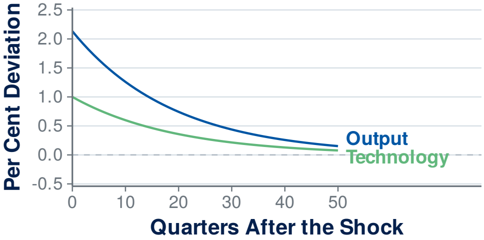

Figure 5: Output against technology after the same shock.

> **Output Peaks on Impact and Has No Hump**
> Output rises 2.1 per cent, then decays. Hours rise 1.7 per cent on impact.

Hours Fall After a Positive Technology Shock

Galí's identified technology shock lowers hours worked, against the model (Galí, 1999):

- **In the Model:** Hours rise, since output rises more than η times consumption.
- **In the VAR:** Hours worked tend to fall after the technology shock.
- **Galí's Reading:** Demand sets short-run output, so technology saves hours.

> **Galí's Finding Is Contested in the Literature**
> Later work specifies hours differently and gets different answers. The question is not settled.

HOURS FIRST-ORDER CONDITION

The Model's Responses Against the Identified VAR

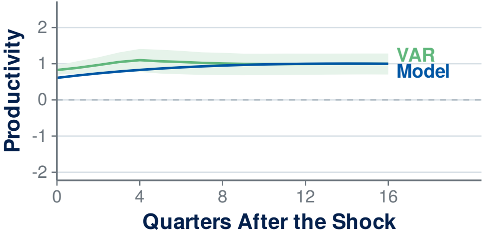

Figure 6: Productivity: model and US VAR, 1948 to 2026, band 90 per cent.

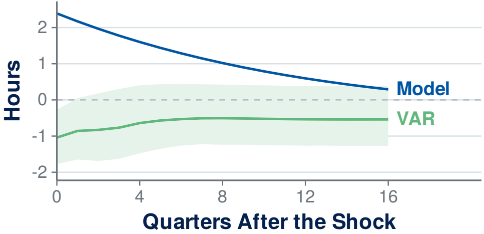

Figure 7: Hours: same shock, both scaled to productivity 1 at 16 quarters.

> **The Two Hours Responses Have Opposite Signs**
> Productivity rises in both. Hours rise in the model and fall in the VAR, significantly on impact.

Rigidities Are the Branch This Course Follows

Two branches of research answer the failures, and the course takes the second:

- **Keep Market Clearing:** Variable utilisation, investment lags and habits tend to strengthen propagation.
- **Add Rigidities:** Sticky prices and wages, which give monetary policy real effects.

> **The Failures Set the Next Four Lectures**
> Weak propagation and no nominal side are the two gaps. Both are filled by Lecture 2.5.

Before Lecture 2.2

1. **Play:**
   Simulate the final system in R, at $\rho = 0.95$ and then at $\rho = 0.5$.
2. **Apply:**
   Derive the first-order conditions from the Lagrangian, without the slides.
3. **Read:**
   Friedman on monetary policy, before Lecture 2.2 (Friedman, 1968).

Next Lecture

Lecture 2.2 builds the supply side the RBC model lacks:

1. **The Expectations-Augmented Curve.**
   Friedman, and the 1970s.
2. **Calvo Pricing.**
   How a firm sets a price that may last years.
3. **The New Keynesian Curve.**
   Derived from Calvo pricing, then solved forwards.
4. **Inflation Persistence.**
   What the forward-looking curve misses.

Thank you

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BACKUP

Appendix

Notation, Part 1

Symbol	What it is	First met
C_t	consumption	What the Social Planner Maximises
N_t	hours worked	What the Social Planner Maximises
U	utility of consumption	What the Social Planner Maximises
V	disutility of hours	What the Social Planner Maximises
β	discount factor	What the Social Planner Maximises
Y_t	output	The Resource and Capital Constraints
I_t	investment	The Resource and Capital Constraints
K_t	capital stock	The Resource and Capital Constraints
A_t	total factor productivity	The Resource and Capital Constraints
δ	depreciation rate	The Resource and Capital Constraints
α	capital share	The Resource and Capital Constraints
A^*	long-run level	Technology Is the Only Source of Shocks
G	expected payoff	Taking the Expectation Through a Derivative
F	payoff in one state	Taking the Expectation Through a Derivative
a_k	state k	Taking the Expectation Through a Derivative

Notation, Part 2

Symbol	What it is	First met
p_k	probability of state k	Taking the Expectation Through a Derivative
$\mathcal{L}$	Lagrangian	The Lagrangian for the Planner's Problem
λ_t	Lagrange multiplier	The Lagrangian for the Planner's Problem
R_{t+1}	gross return on capital	The Marginal Value of a Unit of Capital
Δ	small amount of consumption	Reading the Keynes–Ramsey Condition
η	CRRA curvature	CRRA Utility With Separable Labour
a	disutility	CRRA Utility With Separable Labour
x^*, y^*	chosen point	Approximating a Non-Linear Function by a Line
x_t	log deviation of X_t	Writing a Variable as a Deviation From Steady State
X^*	steady-state value	Writing a Variable as a Deviation From Steady State
ρ	persistence of technology	Lecture 1.1
ϵ_t	technology shock	Lecture 1.1
E_t	expectation at t	Lecture 1.5
Z_t	the solver's vector of variables	Lecture 1.5
X_t	the solver's vector of shocks	Lecture 1.5

Notation, Part 3

Symbol	What it is	First met
K, A, B, H	the solver's four matrices	Lecture 1.5
y_t	log deviation of output	Example One: Substituting, Then Approximating
c_t	log deviation of consumption	Example One: Substituting, Then Approximating
i_t	log deviation of investment	Example One: Substituting, Then Approximating
k_t	log deviation of capital	Example Two: Each Factor Written Around Its Steady State
n_t	log deviation of hours	Example Two: Each Factor Written Around Its Steady State
a_t	log deviation of technology	No Trend Growth Makes Every Steady State Constant
r_t	log deviation of return	The Log-Linear Behavioural Conditions
Y^*	steady-state output	Example One: Substituting, Then Approximating
C^*	steady-state consumption	Example One: Substituting, Then Approximating
I^*	steady-state investment	Example One: Substituting, Then Approximating
K^*	steady-state capital	Example Two: Each Factor Written Around Its Steady State
N^*	steady-state hours	Example Two: Each Factor Written Around Its Steady State
R^*	steady-state return	The Log-Linear Behavioural Conditions
ω	any exponent	A Power of a Level Multiplies Its Deviation

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