LECTURE 2.5

The Smets–Wouters Model

ECON42240 Advanced Macroeconomics

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Where We Are

Last Week's Lecture

1. **From Calibration to a Likelihood.**
   Why estimate, and what a likelihood needs.
2. **Singularity and the Filter.**
   Too few shocks, and the filter as the likelihood.
3. **Priors and Posterior.**
   What a prior adds, and how a sampler draws the posterior.
4. **Judging the Estimate.**
   Reading a posterior, and holding the model against the VAR.

Lecture Plan

1. **The Frictions Block.**
   What Smets and Wouters add, and why.
2. **Shocks and Estimation.**
   Seven shocks, seven series, one posterior.
3. **What the Model Explains.**
   Forecasts, decompositions and responses.
4. **Our Model Against Smets and Wouters.**
   What the simplifications cost.
5. **What It Missed.**
   The model, and 2008.

SECTION 1

The Frictions Block

The Model Central Banks Have Used for Policy

Smets and Wouters estimate this model on US data (Smets & Wouters, 2007):

- **The Authors:** Frank Smets of the ECB, Rafael Wouters of the National Bank of Belgium.
- **The Lineage:** First developed for the euro area, then estimated for the US.
- **Its Use:** Models like it have been used for policy analysis at central banks.

Six Additions Repair Two Earlier Failures

Whelan lists six features that the RBC and New Keynesian models lacked:

Three Real Frictions

Investment adjustment costs, capacity utilisation costs and habit. RBC shocks died out fast; these make variables sluggish.

Two Indexations and New Shocks

Price and wage indexation, and many new autocorrelated disturbances. Indexation supplies the inflation persistence 2.3 missed.

Each Friction Answers a Known Failure

None of them is a new finding about behaviour. Each was added mainly to match persistence.

Seven Blocks, and Whelan's Warning About Them

The log-linearised model has seven blocks: supply, demand, consumption, investment, prices, wages and policy.

> **Quite Ad Hoc, in Many Ways**
> It is "hard to argue these are really micro-founded mechanisms". A friction fitted under one policy regime need not hold under another.

The lecture keeps two of the paper's letters that clash with the module's.

THE PAPER'S LETTERS

Habit Formation Slows Consumption Down

Habit formation subtracts, in red, a fraction of last quarter's consumption inside utility:

$$u\left(C_t - \lambda C_{t-1}\right) \quad \text{replaces} \quad u\left(C_t\right) \tag{1}$$

where:

- C_t: consumption
- u: period utility
- λ: habit parameter

> **Lagged Consumption Enters the Euler Equation**
> Consumption then adjusts gradually. The estimated habit parameter is 0.71.

Adjustment Costs and Tobin's q Govern Investment

Investment rises with Tobin's q, the value of installed capital (q_t), and adjusts slowly (Tobin, 1969):

- **The Adjustment Cost:** Falls on changing investment, which peaks three or four quarters after a shock.
- **What Raises q_t:** Higher expected marginal products of capital, in any later quarter.
- **What Lowers q_t:** Higher expected real interest rates and risk premia.

Capital in Use Adds a Margin Within the Quarter

In logs, capital in use is installed capital plus, in red, the utilisation rate:

$$k_t^s = k_{t-1} + z_t \qquad (2)$$

where:

- k_t^s: capital in use
- k_{t-1}: installed capital
- z_t: utilisation rate

Output Can Rise Before New Capital Arrives
Utilisation rises with capital's marginal product; running harder costs output.

Indexation Adds Lagged Inflation to the Curve

Indexation to past inflation gives the Phillips curve a lag, in red:

$$\pi_t = \pi_1 \pi_{t-1} + \pi_2 E_t \pi_{t+1} - \pi_3 \mu_t^p + \varepsilon_t^p \tag{3}$$

where:

- π_1, π_2, π_3: composite weights
- μ_t^p: price mark-up
- ε_t^p: price mark-up shock

The mark-up is minus real marginal cost, $\mu_t^p = -(mc_t - p_t)$, as in Lecture 2.2.

> **Indexation Updates a Price Without a Reset**
> Most firms index their price to past inflation. Only occasionally does one set an optimal price.

Sticky Wages and the Wage Mark-Up

Wages are set like prices, with a mark-up and indexation of their own:

> **The Wage Mark-Up Is a Gap from the Benchmark**
> The wage mark-up (μ_t^w) is the real wage (w_t) less a benchmark: the marginal rate of substitution (mrs_t).

Sticky wages close the gap gradually, indexed to past inflation.

The Policy Rate Adjusts Gradually Towards a Target

Each quarter the central bank sets the rate part way to a target:

- **The Target:** Inflation with weight 2.03, the output gap with weight 0.08.
- **The Gap's Change:** The change in the output gap, with weight 0.22.
- **The Inertia:** Last quarter's rate, with weight 0.81.

> **A Fifth of the Distance Closes Each Quarter**
> Each quarter the rate closes 19 per cent of its distance to target.

Potential Output Needs a Flexible-Price Economy

The rule's output gap needs a potential output that no series measures:

> **Flexible Prices and Flexible Wages**
> Potential output is output with prices and wages both fully flexible. The model solves that economy alongside.

Lecture 2.3's natural output relaxed sticky prices only, so the two gaps differ.

Our Completed Model, in the Module's Letters

Our model's demand, price and policy rows; the app's stage 2 shows all fourteen:

$$
\begin{aligned}
c_t &= c_1 c_{t-1} + (1 - c_1) E_t c_{t+1} - c_3 \left(r_t - E_t \pi_{t+1} + \varepsilon_t^b \right) \\
\pi_t &= \pi_1 \pi_{t-1} + \pi_2 E_t \pi_{t+1} - \pi_3 \mu_t^p + \varepsilon_t^p \\
r_t &= \rho_i r_{t-1} + (1 - \rho_i) \left(\phi_\pi \pi_t + \phi_x x_t \right) + \phi_{\Delta x} (x_t - x_{t-1}) + \varepsilon_t^r
\end{aligned}
\tag{4}
$$

where:

- c_t: consumption
- c_1, c_3: composite weights
- ε_t^b: risk premium shock
- ε_t^r: policy shock
- x_t: output gap
- $\rho_i, \phi_\pi, \phi_x, \phi_{\Delta x}$: rule weights

What the Lecture Simplifies

This lecture teaches Smets and Wouters' model under the module's assumptions:

- **No Trend Growth:** Every steady state is constant, as in Lectures 2.1 to 2.3.
- **Separable Hours:** Hours leave the consumption equation.
- **The Module's Letters:** n_t, θ_p, θ_w and $\sigma = 1/\sigma_c$. Habit λ is not 2.3's loss weight.
- **Potential Output:** Prices and wages both flexible, a wider gap than Lecture 2.3's.

A section after the results sets our model beside theirs. Whelan's slides 3, 6, 7 and 8 slip in places.

Which Friction Would You Drop First

Six additions were made to a model that already had sticky prices.

Drop One Friction

You must drop one friction from this model. Which one, and what would output's response to a policy shock lose?

Take answers from the room.

Habit and Adjustment Costs Make the Hump

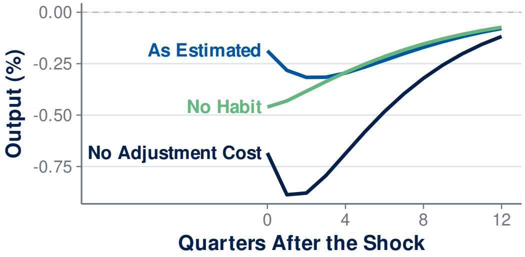

Figure 1: Output after a policy shock, as estimated and with one friction switched off.

> **Two Frictions Delay the Trough**
> Without habit, the trough comes on impact. Without adjustment costs, it is nearly three times as deep.

SECTION 2

Shocks and Estimation

Each Block Carries One of Seven Shocks

This section names the model's seven shocks, then estimates the model:

- **Supply and Spending:** Total factor productivity, and exogenous spending.
- **The Two Euler Equations:** The risk premium, and a shock to investment.
- **Prices, Wages and Policy:** A price mark-up, a wage mark-up, a policy shock.

> **A Shock Absorbs What Its Block Leaves Unexplained**
> Seven blocks, seven shocks. Whatever a block cannot explain is given a shock of its own. Each shock builds on Lecture 1.1's autoregression.

The Risk Premium Shock Reaches Consumption and Investment

The consumption equation carries a disturbance the paper calls a risk premium:

- **The Paper's Reading:** A shift in households' willingness to hold the one-period bond.
- **A Second Reading:** A preference shock over the consumption-saving decision.

> **One Shock Enters Two Equations**
> A higher risk premium lowers consumption and the value of installed capital (q_t).

Exogenous Spending Also Loads on Productivity

Exogenous spending (ε_t^g) follows its own process plus, in red, the productivity innovation:

$$\varepsilon_t^g = \rho_g \varepsilon_{t-1}^g + v_t^g + \rho_{ga} v_t^a \tag{5}$$

where:

- ρ_g: spending persistence
- v_t^g: spending innovation
- ρ_{ga}: productivity loading
- v_t^a: productivity innovation

> **One Process Stands in for Two Components**
> Government spending and net exports share this process; neither is modelled.

A Price Mark-Up Shock Raises Inflation Directly

The inflation equation carries a shock on top of the mark-up itself:

- **What It Does:** A positive shock raises inflation at unchanged marginal cost.
- **What Records It:** No series; it is estimated from the model and the data.

Two Shocks Subtract Their Lagged Innovation

The lagged term, in red, is Whelan's "attempt to get at temporary price level shocks":

$$\varepsilon_t^p = \rho_p \varepsilon_{t-1}^p + v_t^p - \mu_p v_{t-1}^p \tag{6}$$

where:

- ρ_p: price mark-up persistence
- v_t^p: price mark-up innovation
- μ_p: moving-average coefficient

A Sixth of the Shock Survives One Quarter

A quarter on, $\rho_p - \mu_p = 0.90 - 0.74 = 0.16$ of the impact remains. The wage shock has the same form.

Seven Observables Match the Seven Shocks

The paper estimates the model on seven quarterly US series:

- **In Growth Rates:** GDP, consumption, investment, the real wage and the GDP deflator.
- **In Levels:** Hours worked and the federal funds rate.

> **Stochastic Singularity Sets the Shock Count**
> Fewer shocks than series would predict exact relations the data violate. Seven shocks avoid that, as Lecture 2.4 argued.

The Estimation Pipeline Runs at Policy Scale

Estimating this model runs the chain the module built on smaller models:

- **Solve and Rewrite:** The generalised Schur routine, then the state-space form.
- **Filter and Score:** The Kalman filter turns forecast errors into the likelihood.
- **Prior to Posterior:** Bayesian estimation combines the likelihood with the priors.

Table 1A Reports Each Parameter Twice

The paper reports every structural parameter’s prior and posterior side by side:

> **Distance from the Prior Is the Evidence**
> A posterior away from its prior, or narrower, shows what the sample added.

Most Shock Processes Are Highly Persistent

Each shock's persistence is estimated under a beta prior with mean 0.5:

> **Five of Seven Persistences Exceed 0.7**
> Productivity, spending, investment and both mark-ups exceed 0.7. Risk premium and policy fall below 0.25.

The high values come from the data, not the prior.

Price Indexation Sits Furthest from Its Prior

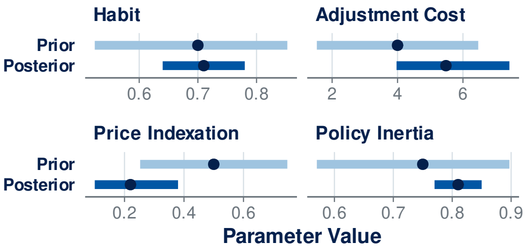

Figure 2: Prior and posterior 90% intervals from the paper's Table 1A, prior means and posterior modes marked.

> **Every Posterior Interval Is Narrower**
> Price indexation's posterior lies wholly below its prior mean of 0.5. Habit's centres on its prior, under half as wide.

Working with a Posterior You Did Not Run

This module ships an estimated posterior rather than asking anyone to produce one:

- **Run Time:** Sampling a model this size outlasts any lecture slot.
- **What You Do With It:** Draw responses and decompositions; compare them with your VAR.

Should the Model Out-Forecast a VAR

A VAR leaves every lag coefficient free; the model ties its parameters together.

> **The Model or a VAR Out of Sample**
> Outside the estimation sample, which should forecast better, the model or a VAR? Why?

Take answers from the room before the next frame.

SECTION 3

What the Model Explains

The Model Beat Both VARs Beyond One Quarter

The paper forecasts outside the estimation sample against two VARs (Sims, 1980):

Table 1: Overall out-of-sample forecast gain over a first-order VAR

Gain on a VAR(1), %	**1q**	**2q**	**4q**	**8q**	**12q**
Bayesian VAR(4)	3.25	0.17	0.51	−4.10	−9.84
Smets–Wouters model	3.06	2.82	6.82	11.50	13.51

Positive Entries Beat the First-Order VAR

The model beats it at every horizon; the Bayesian VAR, beyond one quarter.

Few Free Parameters Make Beating a VAR Hard

The model forecasts with far fewer free parameters than a VAR:

> **Winning with Restrictions Is Evidence**
> Cross-equation restrictions tie the model's parameters together. Winning anyway suggests the restrictions are close to right.

A Shock's Contribution Scales Its Responses by Its Size

Shock j's contribution at horizon h sums its squared responses, times its variance:

$$V_j(h) = \sigma_j^2 \sum_{s=0}^{h-1} \psi_j(s)^2 \tag{7}$$

where:

- $V_j(h)$: contribution of shock j
- h: forecast horizon
- $\psi_j(s)$: response to a unit innovation
- σ_j: shock standard deviation

Drop the Sigma and the Shares Mislead
Unscaled responses treat a sigma of 0.14 like one of 0.52. Lecture 1.2 built this formula for a VAR.

The Variance Share Divides by All Seven Contributions

The share divides shock j's contribution by the total from all seven shocks:

$$\text{share}_j(h) = \frac{V_j(h)}{V_1(h) + V_2(h) + \cdots + V_7(h)} \tag{8}$$

The Seven Shares Sum to One

At every horizon the seven shares sum to one. Whelan's charts stack them to 100%.

Whelan’s GDP Chart Has Different Leaders by Horizon

Whelan’s chart of GDP variance shares has one leader early and another late:

- **One Quarter:** Exogenous spending and the risk premium explain most of it.
- **Forty Quarters:** The wage mark-up explains almost half.

> **A Large Share Is Not an Explanation**
> A large share carries a structural name. Earning the name is Lecture 1.3’s identification problem.

Demand Leads at One Quarter, Mark-Ups at Forty

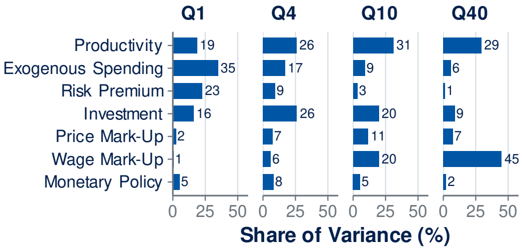

Figure 3: Shares of GDP forecast-error variance at four horizons, our model at the posterior mode.

> **Smets and Wouters' Own Model Gives Whelan's Chart**
> In our model spending and the risk premium hold 57% at one quarter, the wage mark-up 45% at forty. Smets and Wouters' own gives 63% and 44%, as Whelan's chart does.

Inflation and the Policy Rate Have Their Own Leaders

Inflation and the policy rate are explained by different shocks from GDP:

- **Inflation:** The price mark-up leads at one quarter, the wage mark-up at long horizons.
- **The Policy Rate:** The monetary shock, the rule's residual, explains 58% at one quarter on his chart.

Three Shocks Are Grouped as Demand

Whelan groups three shocks as “demand”, in quotation marks:

> **Demand, in Whelan’s Chart**
> Risk premium, exogenous spending and investment shocks each move output and inflation together.

The model draws no such line; the grouping is for readers.

Output's Response to a Policy Shock Is Hump-Shaped

A one-standard-deviation policy shock raises the rate and lowers output and inflation:

- **Output:** Falls to a trough of 0.32 per cent below baseline at quarter two.
- **Inflation:** Falls at once, and by about 0.04 points a quarter later, at a quarterly rate.
- **The Rate:** Rises 0.18 points on impact and is back at zero within seven quarters.

The VAR Shows the Price Puzzle

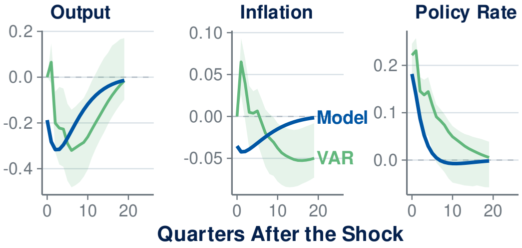

Figure 4: Responses to a policy shock: the model, and a recursive VAR on 1966–2004 data.

> **Output Troughs Agree in Depth**
> Both output troughs are about a third of a per cent below baseline, the VAR's four quarters later. The VAR's ordering fixes output and inflation on impact.

A Wage Mark-Up Shock Lowers Output and Raises Inflation

A wage mark-up shock pushes output and inflation in opposite directions:

- **Output:** Falls for three years before it begins to recover.
- **Inflation and the Rate:** Both rise, so policy tightens as output falls.
- **Its Weight:** It explains almost half of GDP's variance at long horizons.

> **No Series Records the Wage Mark-Up**
> The shock is estimated from the model and the data. Nobody observes it directly.

Each Year's Growth Sums Seven Shock Contributions

A historical decomposition splits GDP growth from its mean (Δy_t) by shock:

$$\Delta y_t = \sum_{j=1}^{7} \Delta y_t^{(j)} + \Delta y_t^{(0)} \tag{9}$$

where:

- $\Delta y_t^{(j)}$: part due to shock j
- $\Delta y_t^{(0)}$: initial-condition part

> **A Historical Decomposition Dates the Shocks**
> A variance decomposition describes an average quarter; this one names the year. It is built from Lecture 1.4's smoothed shocks.

GDP Growth Split into Four Shock Groups

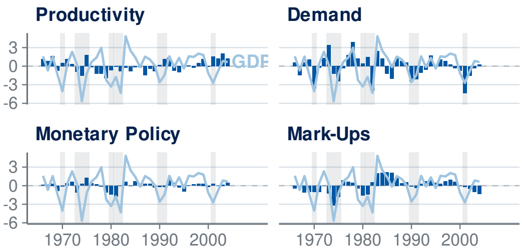

Figure 5: Annual GDP growth from its mean, by shock group, current FRED data, 1966–2004.

> **Demand Shocks Lead Most Recessions**
> Demand leads the falls of 1970, 1982, 1990 and 2001; mark-ups lead 1974. Policy subtracts most in 1980–81.

Which Shock Explains the Last Recession

The decomposition assigns most of 2001's fall to risk premium and investment shocks.

> **Evidence for the Two Labels**
> What outside evidence would convince you those two labels are right?

Take answers from the room.

SECTION 4

Our Model Against Smets and Wouters

Where Our Model Departs from Smets and Wouters'

The app's stage 5 marks each equation of theirs that differs from ours:

Table 2: The equations the app marks as changed or new

Departure	**Equations that differ**
No trend growth	Resource constraint, wage mark-up, every composite weight; trend growth is new
Separable hours	Consumption equation and its weights, wage mark-up
The module's letters	Production, rental rate, price mark-up, policy rule, shocks
Potential output	Potential output, policy rule

Whelan's charts are drawn from their model, not ours.

Our Model's Policy Responses Are Close to Smets and Wouters'

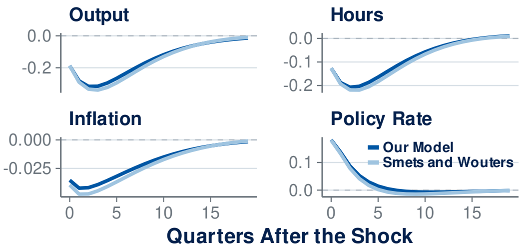

Figure 6: Responses to a one-standard-deviation policy shock: our model and Smets and Wouters' own, at the same posterior mode.

> **Output Bottoms Sooner and Shallower in Ours**
> Output bottoms at 0.32 per cent below baseline at quarter two in our model, 0.34 at quarter three in theirs.

Our Model's Variance Shares Are Close to Smets and Wouters'

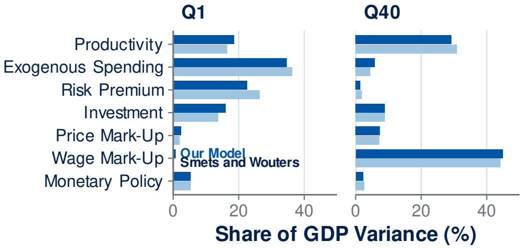

Figure 7: Shares of GDP forecast-error variance at one and forty quarters: our model and Smets and Wouters' own.

> **Demand Explains a Little Less in the Quarter**
> Spending and the risk premium hold 57% at one quarter in ours, 63% in theirs. At forty, the wage mark-up holds 45% and 44%.

SECTION 5

What It Missed

Whelan Offers a Fair List of Weaknesses

This section takes Whelan’s own appraisal of the model just taught:

> **A Fair List Is Not a Prosecution**
> He calls it “a fair list of weaknesses of these models”. He teaches and uses them.

Ad Hoc Mechanisms and Persistent Shocks

The first two weaknesses concern where the model's content came from:

Mechanisms Designed to Fit
Chosen mainly to fit persistence, not from strong belief. Habit is one.

Shocks Left Unexplained
Many, and often highly persistent. The wage mark-up's persistence is 0.97.

A Minimal Financial Sector and Few Policy Tools

The next two weaknesses concern what the model leaves out:

- **Banking and Finance:** A minimal treatment, “still true despite current ongoing work”.
- **Policy Tools and the Accounts:** Very limited modelling of either.

> **The Missing Sector Is the One 2008 Ran Through**
> The model has no credit spread and no bank balance sheet. Lecture 3.1 supplies a financial mechanism.

Rational Expectations and the Lucas Critique Claim

The last two weaknesses target the model's two strongest commitments:

Evidence Against the Pure Assumption
Plenty of evidence says the pure assumption is flawed.

The Verdict Is About a Claim, Not a Model
Claims of stable parameters, and so immunity to the critique, are "silly" (Lucas, 1976).

What a VAR Does Not Impose

Whelan closes the appraisal with three strengths a VAR lacks:

- **Budget Constraints:** Imposed on every agent, in every period.
- **A Consistent Story:** One account of how households and firms behave.
- **Coherent Expectations:** Expectations handled the same way throughout.

> **These Strengths May Help, in His Words**
> He writes they may help forecasting and “what if” analysis. He does not write that they do.

Did DSGE Models Cause the Crisis

Some "new thinking" advocates say DSGE models played a key role in the crisis:

> **Whelan Asks Whether This Is Fair**
> He answers in three parts, conceding some of the charge and rejecting some.

The Laissez-Faire Charge Is Not Really Correct

The charge's first part is that DSGE modelling implies hands-off policy:

- **Whelan's Answer:** The claim is sometimes put forward but is not really correct.
- **His Reason:** New Keynesian models recommend systematic government intervention.
- **This Model's Evidence:** Its policy rule raises the rate with inflation, weight 2.03.

Linearised Models Are Unlikely to Warn of Crises

Whelan concedes the missing financial sector and doubts a repair would warn:

- **The Concession:** The models did not feature banking or financial sectors.
- **The Doubt:** Simple linearised models are very unlikely to capture nonlinear disasters.
- **The Repairs:** Better financial sectors are unlikely to make them crisis-warning tools.

> **A Linearised Response Scales with the Shock**
> Its response to a shock scales with the shock's size. A nonlinear disaster lies outside it.

Central Banks’ Financial Monitors Also Missed the Risks

Whelan’s third answer asks where responsibility for the failure sits:

- **His Premise:** Economics will never be a “one tool for all tasks” business.
- **His Evidence:** Central banks’ financial monitoring departments also missed the risks.

> **Not Responsible for All Failings**
> DSGE modellers “cannot be held responsible for all failings”. The word “all” limits the defence.

Whelan Recommends Sims's Appraisal

Whelan points the class to Chris Sims's INET lecture on DSGE models:

> **A Fair and Balanced Assessment**
> Whelan's words, for an appraisal by the economist who brought VARs to macroeconomics (Sims, 1980).

The appraisal comes from the author of the method this model competes with.

A Model with No Crisis Mechanism

Two facts about this model, both from Whelan's slides, must be read together:

- **What It Did Well:** Its out-of-sample forecasts beat a first-order VAR.
- **What It Contained:** A minimal treatment of banking and finance.
- **What Followed:** A crisis running through the sector the model lacked.

> **Fitting Well Is Not the Same as Being Complete**
> A model can forecast well out of sample yet lack a crisis mechanism.

Before Lecture 3.1

1. **Play.**
   Switch frictions off in the Smets–Wouters app, one at a time.
2. **Apply.**
   Hold the model's responses against your own identified VAR from earlier in the term.
3. **Read.**
   Diamond on banks and liquidity creation, before Lecture 3.1 (Diamond, 2007).

Next Lecture

Lecture 3.1 supplies the financial mechanism this model lacks:

1. **Credit Rationing.**
   Why the loan market need not clear.
2. **The Financial Accelerator.**
   How net worth amplifies a shock.
3. **Runs and Leverage.**
   Diamond–Dybvig, the credit squeeze, and bank leverage.
4. **Capital Regulation.**
   Capital ratios, risk weights and macro-prudential policy.

Thank you

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BACKUP

Appendix

Notation I

Symbol	What it is	First met
π_t	inflation	Lecture 1.2
E_t	expectation formed at date t	Lecture 1.5
C_t	consumption	Habit Formation Slows Consumption Down
λ	habit parameter	Habit Formation Slows Consumption Down
u	period utility	Habit Formation Slows Consumption Down
q_t	value of installed capital	Adjustment Costs and Tobin's q Govern Investment
k_t^s	capital in use	Capital in Use Adds a Margin Within the Quarter
k_{t-1}	installed capital	Capital in Use Adds a Margin Within the Quarter
z_t	utilisation rate	Capital in Use Adds a Margin Within the Quarter
π_1, π_2, π_3	composite weights	Indexation Adds Lagged Inflation to the Curve

Notation II

Symbol	What it is	First met
μ_t^p	price mark-up	Indexation Adds Lagged Inflation to the Curve
ε_t^p	price mark-up shock	Indexation Adds Lagged Inflation to the Curve
mc_t	log marginal cost, as in Lecture 2.2	Indexation Adds Lagged Inflation to the Curve
p_t	log price level, as in Lecture 2.2	Indexation Adds Lagged Inflation to the Curve
w_t	real wage	Sticky Wages and the Wage Mark-Up
μ_t^w	wage mark-up	Sticky Wages and the Wage Mark-Up
mrs_t	marginal rate of substitution	Sticky Wages and the Wage Mark-Up
c_t	consumption	Our Completed Model, in the Module's Letters
c_1	composite weight	Our Completed Model, in the Module's Letters
c_3	composite weight on the real rate	Our Completed Model, in the Module's Letters

Notation III

Symbol	What it is	First met
ε_t^b	risk premium shock	Our Completed Model, in the Module's Letters
ε_t^r	policy shock	Our Completed Model, in the Module's Letters
x_t	output gap, output less potential	Our Completed Model, in the Module's Letters
ρ_i	policy smoothing, as in Lecture 2.3	Our Completed Model, in the Module's Letters
ϕ_π	policy response to inflation	Our Completed Model, in the Module's Letters
ϕ_x	policy response to the gap	Our Completed Model, in the Module's Letters
$\phi_{\Delta x}$	policy response to the gap's change	Our Completed Model, in the Module's Letters
n_t	hours, as in Lecture 2.1	What the Lecture Simplifies
θ_p	Calvo price probability, as in Lecture 2.2	What the Lecture Simplifies
θ_w	Calvo wage probability	What the Lecture Simplifies

Notation IV

Symbol	What it is	First met
σ	intertemporal elasticity, as in Lecture 2.3	What the Lecture Simplifies
σ_c	the paper's inverse elasticity	What the Lecture Simplifies
ρ_g	spending persistence	Exogenous Spending Also Loads on Productivity
v_t^g	spending innovation	Exogenous Spending Also Loads on Productivity
ρ_{ga}	productivity loading	Exogenous Spending Also Loads on Productivity
v_t^a	productivity innovation	Exogenous Spending Also Loads on Productivity
ε_t^g	exogenous spending shock	Exogenous Spending Also Loads on Productivity
ρ_p	price mark-up persistence	Two Shocks Subtract Their Lagged Innovation
v_t^p	price mark-up innovation	Two Shocks Subtract Their Lagged Innovation
μ_p	moving-average coefficient	Two Shocks Subtract Their Lagged Innovation

Notation V

Symbol	What it is	First met
$V_j(h)$	contribution of shock j	A Shock's Contribution Scales Its Responses by Its Size
h	forecast horizon	A Shock's Contribution Scales Its Responses by Its Size
$\psi_j(s)$	response to a unit innovation	A Shock's Contribution Scales Its Responses by Its Size
σ_j	shock standard deviation	A Shock's Contribution Scales Its Responses by Its Size
$\Delta y_t^{(j)}$	part due to shock j	Each Year's Growth Sums Seven Shock Contributions
$\Delta y_t^{(0)}$	initial-condition part	Each Year's Growth Sums Seven Shock Contributions
Δy_t	GDP growth from its mean	Each Year's Growth Sums Seven Shock Contributions
r_t	policy rate, the paper's letter	Reading the Paper Needs Two Letter Swaps
i_t	investment, the paper's letter	Reading the Paper Needs Two Letter Swaps

Reading the Paper Needs Two Letter Swaps

This lecture keeps two of Smets and Wouters's letters that clash with the module's:

- **Used For:** Reading the paper's seven block equations.
- **Why It Matters:** Read i_t as a rate and output would include an interest rate.

> **Investment and the Policy Rate Swap Letters**
> In the paper i_t is investment and r_t the policy rate. Lecture 1.2 fixed i_t as the nominal rate.

Composite weights such as c_1 are functions of deeper parameters, not new ones.

Where Smets and Wouters' Paper and Code Differ

Their published results come from their code, and in three places the code is not the paper as printed:

Table 3: The paper's equations against the authors' code

Where	The paper prints	Their code
Capital accumulation, eq (8)	A factor of about 2 on the investment shock	No such factor
Rental rate, eq (11)	Installed capital k_t	Capital in use k_t^s, as in eq (9)
Risk premium, eqs (2), (4)	The shock in the real rate	The shock scaled by c_3

Their side of the app follows the code. Our model takes eq (8) as derived, with the factor.

Their Published Response Charts Differ from Their Own Code

At the paper's own posterior, their response charts do not match what their code produces:

Table 4: Smets and Wouters' Figures 2, 3 and 6 against their code

Shock	The published charts against their code
Risk premium, investment	Inflation about 25% higher; output, hours and the rate within 5%
Wage mark-up	Early responses about 10% larger
Monetary policy	Inflation about 7% larger; a shallower output trough

Whelan's Response Charts Are the Paper's

Neither their code nor ours can match his charts exactly. His four-quarter variance bars are the paper's five-quarter shares.

What the Model Has Been Checked Against

Table 5: Checks on the app's Smets-Wouters model

Source	**Result**
The paper and its working-paper version	Every equation; Table 1
Their code, run in Dynare	Responses and shares equal to 12 decimals
A re-run of their estimation	Table 1's posterior means
Macro Model Base, Bank of Canada, Jones	Their code's equations; none reproduces the charts
gEcon	Only the 2003 euro-area model
Del Negro and Schorfheide's notes	The eq (8) factor

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