LECTURE 1.4

The Zero Lower Bound and the Liquidity Trap

ECON42550 Macroeconomics

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Where We Are

1. Short-Run Fluctuations

2. NK Building Blocks

3. Long-Run Growth

Last Week's Lecture

Last week expectations stopped being fixed, and the model acquired a time path:

1. **Expectations and Inflation Dynamics.**
   Let expectations follow last year's inflation, and the model gives a path.

2. **The Taylor Principle.**
   How hard the bank must lean for anything to settle.

3. **Scenario Tests.**
   A demand shock, then COVID.

Lecture Plan

1. **Why There Is a Bound.**
   Establish that the policy rate cannot fall far below zero, and why that matters.

2. **The Trap.**
   Show what the model does once the bound binds, and why it is self-reinforcing.

3. **Policy at the Bound.**
   Work through what a central bank can still do when the rate is stuck.

> **Whelan, Chapter 4**
> Reading: Whelan (2023, ch. 4).

SECTION 1

Why There Is a Bound

Can Interest Rates Be Negative?

A negative interest rate on a deposit means you pay the bank for holding your money. That sounds impossible, and it nearly is:

- **Cash Is the Competitor:** A banknote is a claim that pays exactly zero in nominal terms, and anybody can hold one.
- **So There Is an Arbitrage:** If a deposit paid -5%, you would withdraw the money and hold notes instead, and earn 0% rather than -5%.
- **Which Caps How Far the Rate Can Fall:** The central bank can announce whatever rate it likes; it cannot stop people switching into cash.

> **The Floor Is Cash, Not Law**
> The floor under interest rates is not a rule and nobody legislated it. It is the existence of a physical asset that pays zero.

How Far Below Zero?

If the arbitrage were free, the bound would sit at exactly zero. Holding physical cash is not free:

- **Storage, Insurance and Transport:** A pension fund cannot hold €10bn in notes in a drawer, and paying somebody to guard it is a negative return.
- **The True Floor Is Slightly Below Zero:** Rates can go a little negative before cash is worth the trouble. The literature says *effective* rather than *zero*.
- **And Central Banks Went There:** After the financial crisis, several central banks ran policy rates modestly below zero.

> **Why We Still Write Zero**
> The model takes the floor to be zero because the algebra is cleaner. Nothing turns on whether it is 0 or -0.5%. Everything turns on there being a floor.

The Model We Are Adding To

The IS curve from lecture 1.2, unchanged:

$$y_t = y_t^* - \alpha \underbrace{(i_t - \pi_t - r^*)}_{\text{the real interest rate gap}} + \varepsilon_t^y \tag{1}$$

A one-point real interest rate gap cuts output by α points. Trend output (y_t^*) is exogenous. So is the demand shock (ε_t^y).

The Rule We Are About to Break

The Phillips curve and the policy rule, from lectures 1.2 and 1.3, unchanged:

$$\pi_t = \pi_t^e + \gamma(y_t - y_t^*) + \varepsilon_t^\pi \tag{2}$$

$$i_t = r^* + \pi^* + \beta_\pi(\pi_t - \pi^*) \tag{3}$$

The policy rule is what today breaks: it asks for a policy rate (i_t) the bank cannot set.

The Zero Lower Bound

The rule cannot ask for a negative policy rate, so it is truncated at zero:

$$i_t = \max\left[\, r^* + \pi^* + \beta_\pi(\pi_t - \pi^*),\ 0 \,\right] \tag{4}$$

- $\max[a,\ b]$: whichever of a and b is larger

One Max, Two Regimes
When the rule's rate is positive, the bank sets it. The model is then exactly last week's. When the rule's rate is negative, the bank sets zero instead.

The Whole Lecture Is One Inequality
This is the only change to the model all lecture. One inequality, bolted onto one equation. Everything else today is a consequence of it.

What Triggers It

How bad does inflation have to get before the bank runs out of room? Set the rule's rate to zero and solve for π_t:

$$\pi^{ZLB} = \frac{\beta_\pi - 1}{\beta_\pi} \pi^* - \frac{r^*}{\beta_\pi} \tag{5}$$

- π^{ZLB}: the inflation rate at which the rule asks for a rate of zero

A Word in a Superscript Is Not a Star
y_t^* and r^* are resting points, where a variable settles. π^{ZLB} is a threshold the economy crosses on the way somewhere else.

Below the Threshold, the Bound Binds
Inflation below π^{ZLB} and the bank is at the bound. Above it, the model is last week's.

Reading the Trigger

The formula says which economies sit close to the bound. Two of its three parameters are choices the bank makes. The natural rate of interest (r^*) is not a choice. Take illustrative values $\beta_\pi = 1.5$ and $\pi^* = 2\%$, and vary r^*:

- $r^* = 2\%$ gives $\pi^{ZLB} = -0.67\%$. The bank hits the bound only in outright deflation.
- $r^* = 1\%$ gives $\pi^{ZLB} = 0\%$. Zero inflation is enough.
- $r^* = 0\%$ gives $\pi^{ZLB} = +0.67\%$. Inflation below two-thirds of a point puts the bank at the floor, with inflation still positive.

A Lower r^* Means Less Has to Go Wrong

- Every extra point of r^* lowers π^{ZLB} by $1/\beta_\pi$ of a point. So a **falling** natural rate pushes the trigger **up**, towards the inflation rates economies actually run at.
- r^* is unobservable and is estimated: the Holston et al. (2017) series, revised quarterly by the New York Fed, is the standard one, and it fell across advanced economies over the three decades to 2020.

> **A Lower Natural Rate Raises the Trigger**
> A lower natural rate of interest raises the inflation rate at which the bound bites. The lower r^* is, the less has to go wrong.

What Puts a Floor Under the Policy Rate?

Two questions from the first section, answered from memory rather than from notes.

> **Question**
> Why is there a floor under the policy rate, and what raises the inflation rate at which it bites?

Two minutes on your own, then compare with the person beside you.

SECTION 2

The Trap

IS-MP Above the Bound

With $i_t = r^* + \pi^* + \beta_\pi(\pi_t - \pi^*)$, the real interest rate gap $i_t - \pi_t - r^*$ collapses to $(\beta_\pi - 1)(\pi_t - \pi^*)$, so:

$$y_t = y_t^* - \alpha \underbrace{(\beta_\pi - 1)(\pi_t - \pi^*)}_{\text{the real-rate gap the rule delivers}} + \varepsilon_t^y \tag{6}$$

Above the Bound, the Curve Slopes Down

Provided $\beta_\pi > 1$, higher inflation raises the policy rate more than one-for-one. The real interest rate rises, and output falls.

The Branch Below the Bound

At the bound the rule is replaced by a constant, and the IS curve is unchanged:

$$i_t = 0, \qquad y_t = y_t^* - \alpha\left(i_t - \pi_t - r^*\right) + \varepsilon_t^y \tag{7}$$

Zero lands where the policy rate was:

$$y_t = y_t^* - \alpha\left(\boxed{0} - \pi_t - r^*\right) + \varepsilon_t^y \tag{8}$$

> **At Zero, the Rate Is Minus Inflation**
> The real rate is $i_t - \pi_t$; at zero it is $-\pi_t$, and inflation alone sets it.

What the Bound Leaves Behind

Take the zero out and read what is left driving demand:

$$y_t = y_t^* - \alpha \underbrace{\left(-\pi_t - r^*\right)}_{\text{the real-rate gap the bound delivers}} + \varepsilon_t^y \tag{9}$$

The Slope Flips Below the Bound
Above the bound, higher inflation lowers output. Below it, higher inflation *raises* output.

The Kinked IS-MP Curve, Drawn

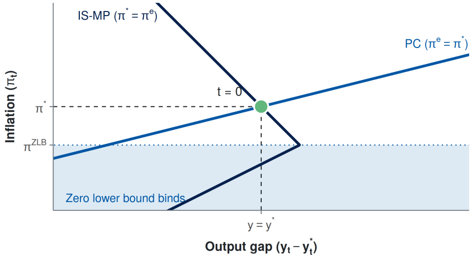

Figure 1: The IS-MP curve slopes down above π^{ZLB} and turns upward below it. The region in which the bound binds is shaded and labelled, and π^{ZLB} is marked on the right axis.

> **The Curve Kinks at the Trigger**
> Above π^{ZLB} the curve slopes downwards and the policy rate is positive. Below the kink the curve slopes upwards, and that region is the trap.

Inflation Dynamics at the Bound

Above the bound, a fall in inflation is self-correcting. The bank cuts the policy rate by more than the fall in inflation, the real interest rate falls, and demand recovers.

At the bound, the same fall in inflation does the opposite:

- Inflation falls, and the policy rate cannot follow, because it is already at zero.
- The real interest rate, $-\pi_t$, therefore **rises**.
- A higher real interest rate cuts demand, so output falls further below trend.
- A larger negative output gap pushes inflation down again, through the Phillips curve.

> **This Is the Liquidity Trap**
> The same feedback that stabilised inflation above the bound now amplifies its fall. That loop is the liquidity trap, and the model produces it unaided.

A Negative Demand Shock

The same shock as last week, $\varepsilon_t^y < 0$, but this time the economy starts at the bound and the policy rate cannot fall.

> **Question**
> Compared with the same shock away from the bound, is the fall in output **smaller**, **the same**, or **larger**?
> And a second one, harder: does inflation end up above or below where it started?

Have a think on your own first, then take ninety seconds with the person beside you.

The Shock, Traced

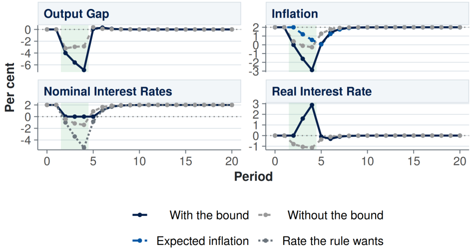

Figure 2: Four time paths after a negative demand shock, with the bound in solid navy against the same shock without it in grey dashed, alongside the rate the rule wants, which runs far below zero while the actual rate stays pinned at zero. The output gap falls about twice as far with the bound, and the real interest rate rises.

> **The Bound Makes the Shock Worse**
> Away from the bound, the rate cut cushions the shock. At the bound nothing does, so output falls further and inflation falls with it. The gap between the two paths is the cost of the constraint.

Why It Gets Worse

So far expected inflation has been fixed at the target. Let it follow realised inflation instead, and the trap deepens:

- Expectations follow recent inflation, so a long low spell lowers π_t^e.
- A lower π_t^e shifts the Phillips curve down, so realised inflation falls further.
- Lower inflation raises the real interest rate, because the policy rate is stuck.
- Output falls further, inflation falls further, and the loop turns once more.

The Expectations Term Does the Damage

The term doing the damage is the expectations term Friedman (1968) put into the Phillips curve. In lecture 1.3 it made policy weaker than it looked; here it makes a shock worse than it looks.

What Stops It

The loop's one exit inside the model is expectations, not the policy rate. If expected inflation (π_t^e) stays at the target, the first arrow never fires. Output stays below trend but stops falling.

Anchored Expectations Prevent the Spiral

Anchored expectations are not a slogan about communication. In this model they separate a recession from a spiral, and the next section holds that anchor.

The Four Steps of the Loop

The loop on "Inflation Dynamics at the Bound" has four steps, and the order matters.

> **Question**
> Write down the four steps that take a fall in inflation into a deeper fall in output, in order.

Two minutes on your own, then compare with the person beside you and fix whichever of you is wrong.

SECTION 3

Policy at the Bound

What Is Left Works on What People Expect

This section asks what a central bank can still do at the bound. Every instrument below works on the real interest rate people **expect**.

> **A Step Beyond the Model as Written**
> At the bound, our IS curve's real interest rate $i_t - \pi_t$ is simply $-\pi_t$. The instruments below act on $-\pi_t^e$ instead. Lecture 2.1 justifies reading the curve forward.

A Taylor Rule at the Bound

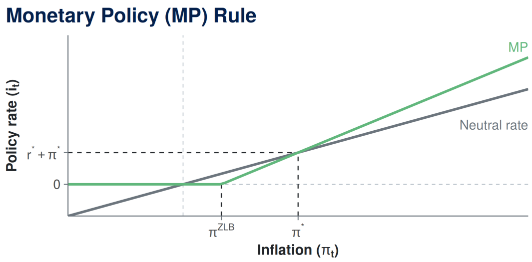

Figure 3: The policy rate against inflation: flat at zero below π^{ZLB}, then rising more than one-for-one above it, with the rule $i = \max[r^* + \pi^* + \beta_\pi(\pi - \pi^*),\ 0]$ written along the line.

> **The Rule Goes Flat at the Trigger**
> To the right of π^{ZLB} the rule is a line of slope β_π. Left of the corner the rule is flat and the policy rate does nothing.

Japan

A liquidity trap in a textbook lasts a few periods. In at least one large economy it lasted decades:

- **What a Long Trap Looks Like:** Policy rates at or near zero not for a quarter or two but for decades; inflation hovering around zero or below; output persistently under trend; and each new shock arriving with the interest rate instrument already spent.
- **Japan Is the Case:** Japanese policy rates reached effectively zero in the 1990s and the economy spent most of the following quarter-century at or near the bound, through several attempts to escape it.

> **Japan Is Not a Footnote**
> Everything in this lecture so far has been drawn for one period. Japan is the reason the model is taught with the bound in it rather than as a footnote.

Thirty Years at the Floor

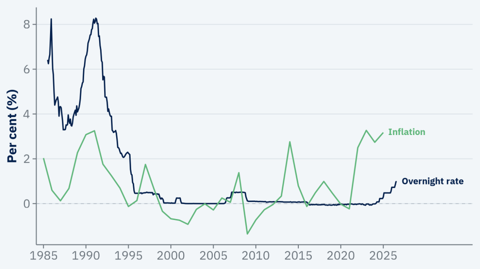

Figure 4: Japanese policy rate and inflation.

> **Zero Inflation Kept the Rate at Zero**
> The policy rate flat along the bottom is the constraint. Inflation crossing zero repeatedly is why the constraint kept binding.

Getting Out: Forward Guidance

Today’s policy rate cannot fall, but demand depends on more than today’s rate:

- **The Instrument:** The bank promises low rates for longer after the bound lifts.
- **Why It Helps Now:** The promise raises expected inflation and lowers today’s real rate.
- **Why It Is Hard:** When recovery comes, the bank will want to break the promise.

> **A Broken Promise Persuades No One**
> A promise the bank will not want to keep is not believed. An unbelieved promise leaves today’s expectations unchanged.

Getting Out: Raising Expected Inflation

Looking forward, the real interest rate at the bound is $-\pi_t^e$. Anything that raises expected inflation lowers it:

- **Raise the Target:** A higher π^* raises π^{ZLB} as well — but only by $(\beta_\pi - 1)/\beta_\pi$ of a point for each point of target, which is less than one. The inflation the bank delivers rises by the full point, so the buffer between the two widens and the bound bites less often. The cost is higher average inflation in the good times too.
- **Promise to Overshoot:** Commit to running inflation above target once the constraint lifts. Expectations then rise today.

> **Both Work Only Through Belief**
> Neither uses today's policy rate. Both work only if people believe a promise the bank will want to break.

Recommending It Is Easier Than Doing It

Bernanke Versus Bernanke
The advice given to Japan in the late 1990s by academic economists — Ben Bernanke, then a Princeton macroeconomist and later chair of the Federal Reserve, among them — was blunt: commit publicly to a higher inflation rate and accept the loss of face. Those same economists in office after 2008 found the commitment far harder to make than to recommend. The difficulty is not intellectual; it is that a central bank's whole institutional reputation is built on promising the opposite.

A Promise Nobody Has to Keep
A promise that is credible only because the person making it will not be there to keep it is not a policy. This is the same credibility problem as forward guidance, one step worse.

Getting Out: Asset Purchases

The bank controls a short-term interest rate, and it is stuck. Long-term rates are not:

- **What It Is:** The bank buys long-dated government bonds with newly created reserves.
- **The Intended Channel:** Buying pushes bond prices up and long-term yields down, so mortgages and corporate debt get cheaper.
- **A Second Channel:** The size of the programme signals how long the bank intends to keep policy loose.

The Instrument Is Now a Quantity

The instrument is no longer a price the bank announces. It is a quantity of bonds bought, recorded on the balance sheet.

The Balance Sheet Became the Instrument

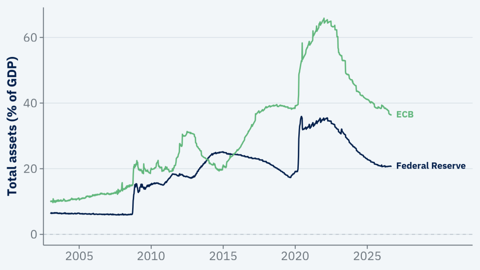

Figure 5: Central bank balance sheets as a share of GDP.

> **The Scale Is the Story**
> Read the level, not the wiggles. After 2008, and again in 2020, the balance sheet replaced the policy rate.

Getting Out: Changing the Framework

Every instrument so far is a promise made in a crisis. A framework is a promise made in advance:

- **Average Inflation Targeting:** Aim at 2% *on average*, so undershooting today obliges the bank to overshoot later.
- **Why It Follows from the Model:** If the bound sometimes binds, inflation averages below target unless the bank plans to overshoot.
- **What It Costs:** A promise to overshoot is awkward to explain when inflation is already high.

> **Expectations, Not Today's Rate**
> Everything in this section works through what people expect, not through what the rate is today. That is what the bound leaves a central bank.

When the Rate Cannot Fall

The model, with one inequality added:

1. **Why There Is a Bound.**
   The policy rate cannot go far below zero, because cash exists.
2. **The Trap.**
   At the bound, falling inflation raises the real interest rate and deepens the slump.
3. **Policy at the Bound.**
   Everything left works through expectations, not through today's rate.

Before Lecture 1.5

1. **Play:** The toy models. Run "Deep Recession at the ZLB" with expectations anchored, then adaptive.
2. **Apply:** Problem set 3, due before Lecture 1.5. It covers today.
3. **Review:** Diamond's Richmond Fed exposition and Romer, chapter 10, on Brightspace, before Lecture 1.5.

One line before you go: what was the muddiest point today?

> **Today Assumed $\beta_\pi > 1$ Throughout**
> Today assumed $\beta_\pi > 1$ every time the IS-MP curve sloped downwards. Lecture 1.3 asked what happens when it does not, and why that condition has a name.

Next Week

Next week the constraint is a bank's balance sheet, not the interest rate:

- **Why Banks Are Fragile:** Three dates, two kinds of depositor, one illiquid project.
- **Runs:** The contract that makes a bank useful has a second, bad equilibrium.
- **The Financial Cycle:** Credit, leverage, and the difference between running out of cash and running out of capital.

Thank you

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BACKUP

Appendix

Notes and Tools

- **Whelan's lecture notes** — the notes for this module. Chapter 4 goes with today.
- **The toy models** — IS-MP-PC with the bound switched on, in your browser, nothing to install.
- **The New York Fed's r^* page** — Holston–Laubach–Williams estimates, revised quarterly. The parameter that decides how often the bound bites.

> **Only Whelan Is Required**
> Whelan is the only required reading.

References

Friedman, M. (1968). The role of monetary policy. *American Economic Review*, *58*(1), 1–17.

Holston, K., Laubach, T., & Williams, J. C. (2017). Measuring the natural rate of interest: International trends and determinants. *Journal of International Economics*, *108*, S59–S75. https://doi.org/10.1016/j.jinteco.2017.01.004

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