LECTURE 3.3

Endogenous Growth

ECON42550 Macroeconomics

Sam Deegan
University College Dublin

Autumn 2026

Where We Are

Last Week's Lecture

Last week we asked what causes growth rather than what merely accounts for it.

1. **Proximate and Fundamental.**
   Separate what *accounts* for growth from what *causes* it.

2. **The Candidates.**
   Take each candidate cause seriously — luck, geography, culture, institutions — and see how far it gets.

3. **Institutions.**
   Make the case for institutions, and confront the identification problem head on.

Lecture Plan

1. **Ideas Are Different.**
   Why technology cannot be left outside the model, and what kind of economic good an idea is.

2. **The Romer Model.**
   Growth built out of a research sector.

3. **Semi-Endogenous Growth.**
   The model's sharpest prediction fails in the data. Repair it, and the policy conclusion changes.

4. **Gordon and the Future of Growth.**
   The case that the great century of innovation was a one-off, and where AI enters it.

Reading: Whelan (2023, ch. 12).

SECTION 1

Ideas Are Different

The Hole in Solow

This section establishes why technology cannot be left outside the model.

$$g_y = g_A \tag{1}$$

- g_y: the long-run growth rate of output per worker
- g_A: the growth rate of technology

Everything Solow explains — saving, depreciation, population growth, capital — sets the path's **level**. The **slope** is g_A, handed to the model from outside. Solow never explains why A grows. That hole is the entire long run.

> **What 3.1 Called g Is g_A from Here On**
> Same object, longer name: the growth rate of labour-augmenting technology. Solow's dilution term $(n + g + \delta)$ is now written $(n + g_A + \delta)$.

What “Endogenous” Means

Exogenous
Determined outside the model. The model takes the variable as given and says nothing about where it came from.

Endogenous
Determined inside the model, by the choices and constraints the model describes. The model has to explain it.

In Solow, A is exogenous: it grows at a rate written on the page. The research programme of the late 1980s and 1990s — Romer (1990) above all — was to move A from the first box to the second, by writing down who produces ideas and what they are paid for doing it. “Endogenous growth” is a claim about the *model*: technology has always been endogenous in the world.

Ideas as Economic Goods

To bring A inside the model, first ask what sort of thing an idea **is**. Economists sort goods on two properties, and every good sits somewhere on both.

Rivalry
A good is **rival** if one person's use of it stops another person using it. A litre of diesel, an hour of a nurse's time, a seat on a plane.

Excludability
A good is **excludable** if the owner can stop others using it. A fenced field is excludable; the light from a lighthouse is not.

An idea is **non-rival** and only **partially excludable**. The formula for a vaccine, the design of a semiconductor, double-entry bookkeeping: my using it takes nothing away from your using it, and keeping you out requires a patent, a trade secret or a head start — all of which are leaky and all of which expire.

Non-Rivalry Has Teeth

Non-rivalry is the reason growth can go on for ever:

- **A Rival Input Is Divided:** ten factories need ten sets of machines.
- **A Non-Rival Input Is Not:** one design sits in every factory, at zero marginal cost.
- **Doubling Output Twice Over:** twice the machines and workers, or one better design.

> **Ideas Never Get Spread Thin**
> Each extra machine is spread over the same workforce, so it adds less than the last.

SECTION 2

The Romer Model

Ideas Enter as a Third Input, Not as a Shifter

This section builds growth out of a research sector that produces designs.

$$Y = K^{\alpha}\big(\underbrace{AL_Y}_{\text{effective labour}}\big)^{1-\alpha} \tag{2}$$

- L_Y: labour employed making goods

The function is Solow's, with A now a stock the model must produce.

Constant Returns to What Is Rival, Increasing to Everything

Count returns to scale twice over, on the same function:

- **Constant in the Rival Inputs:** a second identical factory with a second identical workforce gives a second identical output.
- **Increasing in All Three:** A need not be duplicated, so doubling K, L_Y *and* A more than doubles Y.

Non-Rivalry Is What Makes Returns Increasing
$F(2K, 2L_Y, A) = 2Y$, but $F(2K, 2L_Y, 2A) > 2Y$. Doubling output needs no second copy of A.

Why Perfect Competition Cannot Survive This

Increasing returns is not a technicality. It destroys the market structure every other model in this course has assumed:

- Under constant returns to the rival inputs, marginal products **exhaust output**.
- Inventing A costs money once, up front, before a single unit is sold.
- Marginal-cost pricing never covers a fixed cost, so price-takers do no research.

> **Monopoly Rents Pay for Research**
> Romer's answer is to drop the competitive benchmark: ideas are sold by firms with **market power**, and those rents pay for the research.

What Non-Rivalry Does to Returns to Scale

Write your answer down without looking, then compare with the person beside you.

> **Question**
> What does it mean for a good to be non-rival, and what does non-rivalry do to returns to scale?

Then take answers from the room.

The R&D Sector

$$\dot{A} = \bar{\delta}\, L_A \underbrace{A^{\phi}}_{\text{the standing-on-shoulders term}} \tag{3}$$

- $\dot{A}$: new ideas produced per unit of time
- $\bar{\delta}$: research productivity
- L_A: labour employed in research
- ϕ: how strongly the existing stock helps

> **A Dot Is a Rate of Change**
> $\dot{X}$ means dX/dt: the change in X per unit of time. We are in continuous time, as in 3.1, so a growth rate is $\dot{X}/X$.

Standing on Shoulders, or Fishing Out

ϕ says what the stock of ideas does to new research — not the ϕ of 1.4 or 2.1:

- $\phi > 0$: **standing on shoulders.** Past discoveries raise research productivity.
- $\phi = 0$: **neither effect.** The stock leaves research productivity unchanged.
- $\phi < 0$: **fishing out.** The easy ideas went first, so each new one costs more.

> **The Boundary That Matters Is $\phi = 1$**
> Jones (1995) frames fishing out as $\phi < 1$, not $\phi < 0$. Treat the three cases above as intuition for the sign, and remember $\phi = 1$.

The Allocation of Labour

$$L = L_A + L_Y, \qquad L_A = s_A L, \qquad \frac{\dot{L}}{L} = n \tag{4}$$

- s_A: the share of the workforce doing research

More Researchers Means Less Output Now

Every researcher is a worker not making goods. Raising s_A buys more ideas and gives up current output, and that trade-off is the whole of research policy.

The Growth Rate of Technology

Divide the ideas equation through by A:

$$g_A \equiv \frac{\dot{A}}{A} = \bar{\delta}\, s_A L\, A^{\phi-1} \tag{5}$$

Two forces, pulling opposite ways:

- **More Researchers Raise g_A:** g_A rises with s_A and with L.
- **A Bigger Stock Lowers g_A:** when $\phi < 1$ the same effort yields less growth.

Growth Is an Outcome, Not a Parameter
Growth is an outcome of how many people are put to work looking for ideas.

The Dynamics of g_A

Take logs of $g_A = \bar{\delta} s_A L A^{\phi-1}$ and differentiate in time:

$$\frac{\dot{g}_A}{g_A} = n + \underbrace{(\phi - 1)\, g_A}_{\text{the diminishing-returns drag}} \tag{6}$$

Multiplying through gives the change in the growth rate itself ($\dot{g}_A$):

$$\dot{g}_A = g_A \left[n + (\phi - 1)\, g_A \right] \tag{7}$$

Population Growth Offsets a Shrinking Return

A growing population raises g_A; a growing stock of ideas lowers it when $\phi < 1$. Suppose they balance: section 3 says when, and at what value.

The Complete Model

Four blocks, three of them already on the table:

$$Y = K^{\alpha}(AL_Y)^{1-\alpha}, \qquad \dot{A} = \bar{\delta} L_A A^{\phi} \tag{8}$$

$$\dot{K} = sY - \delta K, \qquad L = L_A + L_Y, \quad \frac{\dot{L}}{L} = n \tag{9}$$

> **Two Pairs, and a Bar Is the Only Thing Separating One**
> s is the fraction of output saved; s_A the fraction of labour in research. δ is depreciation from 3.1; $\bar{\delta}$ is research productivity.

Two Stocks Accumulate, and Only One of Them Wears Out

Put the two accumulation equations side by side. They are not the same kind of object:

- **Capital:** $\dot{K} = sY - \delta K$. It depreciates, and it is shared out among workers — which is where Solow's diminishing returns came from.
- **Ideas:** $\dot{A} = \bar{\delta} L_A A^{\phi}$. Nothing wears out and nothing is shared out: the same A sits in every sector at once.

Ideas Accumulate Without Ever Depreciating

Capital accumulates exactly as it did in Solow. What is new is a second stock beside it that never depreciates and never thins.

The Balanced Growth Path

Write capital per unit of effective labour, and its law of motion:

$$\tilde{k} = \frac{K}{AL} \tag{10}$$

$$\dot{\tilde{k}} = s\,\tilde{k}^{\alpha}\,\underbrace{(1 - s_A)^{1-\alpha}}_{\text{the goods-sector share}} - (n + g_A + \delta)\,\tilde{k} \tag{11}$$

A Tilde Means per Unit of Effective Labour

$\tilde{k}$ divides capital by AL, not by L: two growing variables, one steady ratio.

Dilution Now Grows With g_A Too

Same shape as in 3.1, except dilution carries g_A and researchers make no goods.

What Grows, and at What Rate

Solve the production function for output per worker:

$$\frac{Y}{L} = A\,\tilde{k}^{\alpha}(1 - s_A)^{1-\alpha} \tag{12}$$

On the balanced growth path $\tilde{k}$ and s_A are constant, so only A grows. Whether that growth rate is also a *policy variable* is section 3's question, and the answer is less comfortable than it looks.

> **The Growth Rate Is No Longer Assumed**
> Output per worker grows at g_A and at nothing else. Capital sets the level of the path; the slope now comes from a sector the model describes.

The Ideas Production Function, from Memory

Sketch it without looking, then compare with the person beside you.

> **Question**
> Write down the ideas production function, label every symbol, and say in one line what ϕ does.

Then take answers from the room.

SECTION 3

Semi-Endogenous Growth

The Scale Effect

This section confronts the model's sharpest prediction with the data.

Take $\phi = 1$, the case the Romer model is usually taught in. Then $A^{\phi-1} = 1$ and the growth rate collapses to

$$g_A = \bar{\delta}\, s_A L \tag{13}$$

The stock of ideas drops out entirely. What is left is the **number of people doing research**.

> **Bigger Economies Should Grow Faster**
> The growth rate rises with the *size* of the economy: twice the researchers, twice the growth rate, for ever.

The Scale Effect Fails

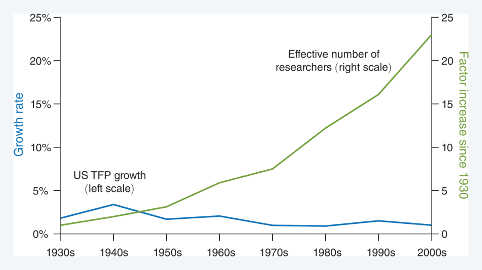

Figure 1: US total factor productivity growth and the effective number of researchers, by decade, 1930s to 2000s.

Researchers Rise, TFP Growth Does Not

Researchers in R&D rise steeply throughout; measured TFP growth is flat or falling. If the scale effect were real, the two series would rise together.

The Phase Diagram

With $\phi < 1$ now the live case, plot $\dot{g}_A$ against g_A:

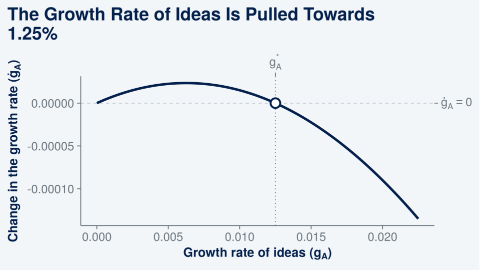

Figure 2: The change in the growth rate of ideas, $\dot{g}_A$, against its level: a concave curve, positive below g_A^* and negative above it, crossing zero at the marked value $g_A^* = 0.0125$.

> **Only the Right-Hand Crossing Is Stable**
> g_A rises where the quadratic sits above the axis and falls where it sits below.

Long-Run Growth Settles at $n/(1-\phi)$

Set $\dot{g}_A = 0$ in the dynamic equation:

$$g_A \left[n + (\phi - 1)\, g_A \right] = 0 \tag{14}$$

Either $g_A = 0$, or the bracket vanishes at the balanced-growth rate (g_A^*):

$$g_A^* = \frac{n}{1-\phi} \tag{15}$$

Growth Does Not Depend on the Research Share

Long-run growth depends on the population growth rate and on how severe the diminishing returns to ideas are. Not on s_A, and not on $\bar{\delta}$.

Half the Hole Is Filled, and the Other Half Is Now *n*

- **What Romer Keeps:** Technology is still produced inside the model, by people the model counts and pays. Ideas are not handed down from outside.
- **What Romer Loses:** The long-run growth *rate* is pinned by n, and the model has nothing to say about where n comes from.

Now n Is Handed In from Outside

Solow handed you g_A from outside; this hands you n from outside instead. The mechanism in between is now written down, which is a real advance.

Population Growth Is What Keeps g_A Positive

Why growth survives at all, once ideas get harder to find:

- Ideas get harder to find as the stock grows, so a **constant** number of researchers produces a **falling** growth rate.
- A **growing** population means a growing number of researchers, which offsets that decline.
- Growth survives in the long run only because the research workforce keeps expanding. The two effects balance at $n/(1-\phi)$.

> **A Prediction Nobody Wants**
> Set $n = 0$ with $\phi < 1$ and $g_A^* = 0$: the economy stops growing. With fertility below replacement across the rich world, that is not a corner case.

Two Kinds of Policy, and Only One Bends the Path

Lecture 3.1 drew this line in the Solow model. The same two words decide what a research subsidy can do here:

A Growth Effect Compounds Forever
A policy that changes the slope of the path. Output per worker is permanently *further ahead* every year than it would have been.

A Level Effect Never Compounds
A policy that shifts the path up once. Output per worker is permanently higher, by a fixed proportion, and then grows at the same rate as before.

A Research Subsidy Is a Level Effect

On the balanced growth path g_A equals $n/(1-\phi)$. Put that into $g_A = \bar{\delta} s_A L A^{\phi-1}$ and solve for the stock:

$$A \;\propto\; \left(\bar{\delta}\, s_A L\right)^{\frac{1}{1-\phi}} \tag{16}$$

Everything a research policy can touch — $\bar{\delta}$, s_A — is inside that bracket, and the bracket sets A, not the rate at which A grows.

A Subsidy Is Not a Growth Policy

Doubling the research share raises the technology path permanently, by a factor $2^{1/(1-\phi)}$, against the output given up through $(1-s_A)$.

Strictly Endogenous Growth

Research policy raises growth for ever only at exactly $\phi = 1$. That value goes into the drag term of the dynamic equation, and kills it:

$$\dot{g}_A = g_A \left[n + (1 - 1)\, g_A \right] = n\, g_A \tag{17}$$

Two cases, and only two:

- $n > 0$: g_A grows without bound and output accelerates, which no data shows.
- $n = 0$: $\dot{g}_A = 0$ at any level, so a subsidy raises $g_A = \bar{\delta} s_A L$ permanently.

> **Strictly Endogenous Growth Needs $\phi = 1$**
> Below $\phi = 1$, growth reverts to $n/(1 - \phi)$ whatever research policy does; above it the model explodes. That knife-edge is the standard objection.

Two Models, Two Policy Conclusions

One parameter separates strictly endogenous from semi-endogenous growth, and it decides whether growth policy exists.

1. **Strictly endogenous, $\phi = 1$ with $n = 0$.**
   Raising s_A raises the growth rate for ever, so growth is a policy choice.

2. **Semi-endogenous, $\phi < 1$.**
   Raising s_A lifts the technology path, but population growth fixes the growth rate.

3. **What is common to both.**
   Both lift the path permanently; they differ on whether the gain compounds.

> **ϕ Is Not Directly Observable**
> Which one you believe is an empirical claim about ϕ, so the argument is live.

Do We Do Too Much Research, or Too Little?

Is the market's s_A the right one? Four wedges separate the private from the social return, and they pull both ways:

- **Standing on Shoulders:** future researchers gain, and nobody pays — **too little**.
- **Consumer Surplus:** an innovator never captures the whole value — **too little**.
- **Duplication:** two labs race for the same patent; one run was enough — **too much**.
- **Business Stealing:** profit is taken from an incumbent, not created — **too much**.

> **Theory Does Not Sign the Sum**
> Whether an economy under-invests or over-invests in research is an empirical question, and answering it means measuring the four wedges separately.

SECTION 4

Gordon and the Future of Growth

Gordon: Was the Great Century a One-Off?

Gordon (2012) argues that 1870 to 1970 was not normal. Electricity, the internal combustion engine and plumbing raised productivity once, enormously.

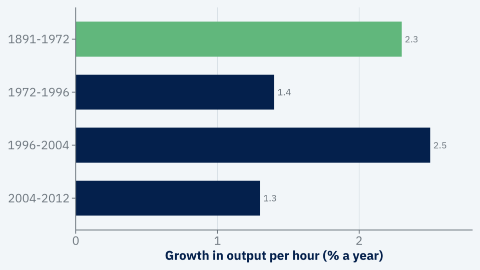

Figure 3: Growth in US output per hour, by sub-period, from the late nineteenth century onward.

> **The Mid-Century Bars Stand Apart**
> The tall mid-century bars are the anomaly; the short ones on either side are the norm.

Gordon’s Thought Experiment

Gordon puts the argument as a choice between two bundles of inventions.

> **Keep Plumbing or Keep the Internet**
> You keep every invention up to a chosen cut-off date, and give up everything invented after it. One bundle keeps the household technologies of the second industrial revolution; the other keeps the digital ones.

Ninety seconds on your own, then ninety with the person beside you. Ask how large the gap between the two bundles is, not which bundle wins.

The Headwinds

Gordon’s six headwinds hold back output per head whatever innovation does.

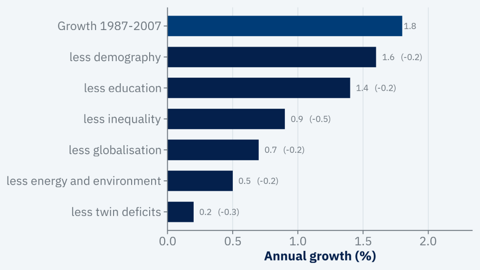

Figure 4: Gordon’s own exercise in subtraction: the growth rate left after each headwind.

> **Headwinds Change the Level, Not g_A**
> Demography, education, inequality, globalisation, energy and debt lower measured output per head, not g_A. The steps are illustrative arithmetic.

Secular Stagnation

Gordon's supply-side pessimism has a demand-side counterpart: desired saving persistently exceeds desired investment. The real interest rate that would clear the two is then very low, and possibly negative:

- Slower population growth and slower productivity growth both **reduce the demand for investment**, because both lower the capital stock the economy needs to equip and re-equip each year.
- A lower natural rate of interest (r^*) leaves less room to cut before the zero lower bound — the constraint of lecture 1.3, from the long-run side.

Estimates of the natural rate of interest, Holston et al. (2017), are the evidence usually put behind this argument. They are the same series we used in 1.1.

Artificial Intelligence and the Ideas Equation

Gordon's argument now runs through AI, which enters the **ideas** equation, not goods:

- Does AI raise $\bar{\delta}$ — the same researchers, more ideas per year?
- Or does AI raise ϕ, making existing knowledge more useful to researchers?

> **Only ϕ Changes the Long-Run Growth Rate**
> A bigger $\bar{\delta}$ is a **level effect** under $\phi < 1$, however large.

Where Growth Comes From

Solow left technology outside the model. Romer brings it in:

1. **Ideas are different.**
   Non-rivalry forces increasing returns and rules out the perfectly competitive benchmark.
2. **The Romer model.**
   Research produces ideas, so growth depends on how many people look for them.
3. **Semi-endogenous growth.**
   The scale effect fails in the data, and the repair changes the policy conclusion.
4. **Gordon and the future of growth.**
   The great century may have been a one-off, and AI is the open question.

Before Lecture 3.4

1. **Play:** the Romer app. Set $\phi = 1$, then $\phi = 0.5$, and watch what raising the research share does to the growth rate in each.
2. **Apply:** final revision. No problem set today — Set 6 was due before this lecture and is the last of the six.
3. **Review:** Aghion's Nobel lecture, on Brightspace. It is the primary source for next week and it is short.

Past papers put an exponent λ on L_A — the elasticity of new ideas to researchers, set to one all lecture — so their answer is $\lambda n/(1-\phi)$.

> **Question**
> One line before you go: what is the muddiest point from today?

Next Week

Today's ideas are produced by researchers and paid for by monopoly rents.

> **Growth as Firms Destroying Each Other**
> Lecture 3.4 asks who those monopolists are. Business stealing, one of today's four wedges, is what drives growth there.

Thank you

sam.deegan@ucdconnect.ie
sam-deegan.com

BACKUP

Appendix

Notes and Tools

- **Whelan's lecture notes —** the notes for this module. Chapter 12 goes with today.
- **The toy models —** the Romer app carries today's dynamics, in your browser, nothing to install.

Whelan is the only required reading.

References

Holston, K., Laubach, T., & Williams, J. C. (2017). Measuring the natural rate of interest: International trends and determinants. *Journal of International Economics*, *108*, S59–S75. https://doi.org/10.1016/j.jinteco.2017.01.004

Whelan, K. (2023). *Lecture notes on macroeconomics*. University College Dublin. https://www.karlwhelan.com/Macro2/Whelan-Lecture-Notes.pdf